SUMMIT MATH

Learn at your **OWN** pace.

ADVANCED ALGEBRA & TRIGONOMETRY SERIES

BOOK 2

UNIT 3: **QUADRATIC FUNCTIONS, EQUATIONS & INEQUALITIES**

UNIT 4: **CIRCLES & NONLINEAR SYSTEMS**

INTRODUCTION

Learning math through Guided Discovery:
Guided Discovery is designed to help you experience a sense of discovery as you learn each topic.

Why this curriculum series is named Summit Math:
Learning through Guided Discovery is like hiking. Learning and hiking both require effort and persistence and people naturally move at different paces, but they can reach the summit if they keep moving forward. Whether you race rapidly through the book or step slowly through each scenario, this textbook is designed to keep advancing your learning until you reach the summit.

Guided Discovery Scenarios:
The Guided Discovery Scenarios in this book are written and arranged to show you that new math concepts are related to previous concepts you already know. Try each scenario on your own first, check your answer when you finish, and then fix any mistakes, if needed. Making mistakes and struggling are essential parts of the learning process.

Homework & Extra Practice Scenarios:
After you complete the scenarios in each Guided Discovery section, extra practice will help you develop better memory of each topic. Use the Homework & Extra Practice Scenarios to improve your understanding and to increase your retention.

The Answer Key:
The Answer Key is included to give you teacher-like guidance. When you finish a scenario, you can get immediate feedback. If the Answer Key does not help you fully understand the scenario, try to get additional guidance from another student or a teacher.

Find more resources at:
www.summitmath.com

GUIDED DISCOVERY SCENARIOS

The Guided Discovery Scenarios are designed to help you experience a sense of discovery as you learn. Explanations are brief and carefully timed to allow you to build your learning at a comfortable pace. The Answer Key supports your learning by giving you immediate feedback and helping you understand each step before moving on.

As you complete each scenario, follow these steps.

Step 1: Try the scenario.
Read the scenario carefully and try to work through it. Be willing to struggle.

Step 2: Check the Answer Key.
The Answer Key can guide you through the scenario, show you new ideas, or help you find mistakes in your steps.

Step 3: Fix your mistakes, if needed.
Mistakes are learning opportunities. If your result does not match the Answer Key, check your work and look for errors. If you still need guidance, seek help from a classmate or teacher.

When you are ready, try the next scenario and repeat this cycle.

SAMPLE PAGES

CONTENTS

Unit 3

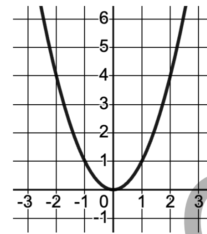
Quadratic Functions, Equations & Inequalities

Section 1

Quadratic Equations & Imaginary Numbers

Use this page for taking notes or anything else that helps you learn.

Methods for Solving Quadratic Equations



The graph of a quadratic function is a parabola. Consider the parabola to the right, formed by $y = x^2$. The function has a y-value of 4 at two x-values: 2 and -2. The graph shows why the equation $x^2 = 4$ has two solutions: $x = \pm 2$.

Quadratic equations can be as simple as $x^2 = 4$ and as complex as $5x^2 + 3x = 2$. They can be solved using different methods.

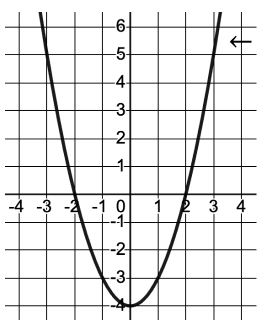
1. Some quadratic equations can be solved by factoring. Use factoring to solve each equation.

   a. $x^2 - 7x + 10 = 0$ b. $x^2 - 36 = 0$ c. $6x^2 - 19x + 3 = 0$

2. Solve this equation in 2 ways, by graphing and then factoring. To solve by graphing, make the 2 sides of the equation separate functions and find the x-values where they intersect.

   a. graphing

   $x^2 - 4 = 5$



$\leftarrow y = x^2 - 4$

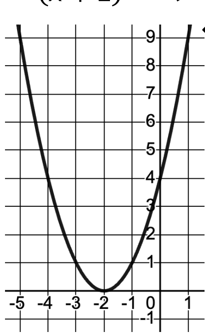
   b. factoring

   $x^2 - 4 = 5$

3. Solve this equation in 2 ways, by graphing and then by taking the square root of both sides.

   a. graphing

   $(x + 2)^2 = 9$



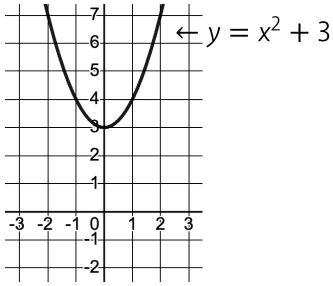
$\leftarrow y = (x + 2)^2$

   b. using square roots

   $(x + 2)^2 = 9$

4. Solve this equation in 2 ways, by graphing and then by taking the square root of both sides.

a. graphing
$x^2 + 3 = -1$



b. using square roots
$x^2 + 3 = -1$

5. In the previous scenario, the graphing method shows the equation $x^2 + 3 = -1$ has no solutions. The function $y = x^2 + 3$ never equals -1....or does it? If $x^2 + 3 = -1$, then $x^2 = -4$.

a. Is there a number that equals -4 after it is squared?

b. Read the following statement as well as you can.

The square root of a negat$\sqrt{-1}$ve number $\sqrt{-1}$s an $\sqrt{-1}$mag$\sqrt{-1}$nary number.

Imaginary Numbers

6. What expression do mathematicians use to represent $\sqrt{-1}$?

You now see that $\sqrt{-1}$ is called i, but you may not know why. The numbers on a typical number line are real numbers. An expression like $\sqrt{-4}$ is not on the real number line so it is defined as imaginary. Since $\sqrt{-4}$ is $\sqrt{4}\sqrt{-1}$, then $\sqrt{-4}$ is $2i$. The $\sqrt{-1}$ is the imaginary part, so i is the "imaginary number."

7. What is the value of each expression shown?

a. $\sqrt{-9}$

b. $\sqrt{-8}$

c. $\sqrt{-45}$

8. The value of $\sqrt{1}$ is 1, because $1^2 = 1$. Since the value of $\sqrt{-1}$ is i, what is the value of i^2?

9. To see why $i^2 = -1$, consider what you already know about square roots. The value of $\sqrt{9}$ is 3 because 3^2 is 9. Continue this logic below by filling in the blank spaces.

a. $\sqrt{49}$ is 7 because 7^2 is _____.

b. Since $\sqrt{\frac{4}{9}} =$ _____, then $(\quad)^2 =$ _____.

c. Since $\sqrt{24} = 2\sqrt{6}$, then $(\quad)^2 = 24$.

d. And finally, since $\sqrt{-1} =$ _____, then $(\quad)^2 =$ _____.

10. Simplify each expression below.

a. $(2i)^2$

b. $(-2i)^2$

c. $\left(i\sqrt{3}\right)^2$

11. The quadratic equation $x^2 = 5$ has 2 solutions: $x = \pm\sqrt{5}$. Solve each equation shown.

a. $x^2 = -9$

b. $x^2 = -12$

c. $(x - 7)^2 = -2$

Exponents of *i*

12. Now that you know $i \cdot i = -1$, consider the values of i^3, i^4, i^5, etc. Notice how a pattern develops when you simplify each value shown.

a. $i^1 = i$

$i^2 =$

$i^3 = i^2 \cdot i =$

$i^4 = i^2 \cdot i^2 =$

b. The expression i^5 can be simplified with several methods:

$i^5 = i^4 \cdot i =$

$i^5 = i^3 \cdot i^2 =$

$i^5 = i^2 \cdot i^2 \cdot i =$

13. Use one of the methods in the previous scenario to find the simplest form of each expression below.

a. i^6 b. i^7

14. The expression i^N can be simplified and written as either i, $-i$, 1 or -1. Simplify each expression.

a. i^2 b. i^3 c. i^4 d. i^5

15. Write the simplest form of each expression below.

a. i^8 b. i^9

16. Use the pattern you found in the previous scenarios to write the simplest form of each expression.

i^1	i^5	i^9
i^2	i^6	i^{10}
i^3	i^7	i^{11}
i^4	i^8	i^{12}

17. Simplify each expression shown.

a. i^{40} b. i^{102} c. i^{63}

18. The pattern with exponents of i applies to negative exponents as well: $i^8 = i^4 = i^0 = i^{-4} = i^{-8}$, etc. Use this pattern to simplify each expression. The first one is started for you.

a. $i^{-1} = i^{-4} \cdot i^3$
$= 1 \cdot -i$
$=$

b. $i^{-2} = i^{-4} \cdot i^2$
$=$

c. i^{-3}

d. i^{-4}

Section 2

Completing the Square & The Quadratic Formula

Use this page for taking notes or anything else that helps you learn.

Completing the Square

Some quadratic equations can be solved by factoring, such as $x^2 - 2x - 3 = 0$. When factoring is not an option, there are two methods you can use: Completing the Square and the Quadratic Formula. You may have learned these in a previous lesson, but it will help to review them now.

1. To use the method of Completing the Square, you need to remember the relationship between the coefficients of a perfect square trinomial and its factors. Factor each expression below.

   a. $x^2 + 6x + 9$

   b. $x^2 - 10x + 25$

2. Fill in the blanks with values that make each pair of expressions equal.

   a. $(x + \_\_\_\_)^2 = x^2 + 8x + \_\_\_\_\_$

   b. $(x - \_\_\_\_)^2 = x^2 - \_\_\_\_\_x + 49$

3. Fill in each blank with a number that makes the expression a perfect square trinomial. Factor the trinomial to confirm its factored form is $(x + \_\_\ )^2$ or $(x - \_\_\ )^2$.

   a. $x^2 + 12x + \_\_\_\_\_$

   b. $x^2 - 18x + \_\_\_\_\_$

> If "c" is $\left(\frac{b}{2}\right)^2$, then $x^2 + bx + c$ is a perfect square trinomial and it can factored as $\left(x + \frac{b}{2}\right)^2$.

4. First, complete each trinomial to make it a perfect square. Then, write it in factored form.

   a. $x^2 + x + \_\_\_\_\_$

   b. $x^2 - 5x + \_\_\_\_\_$

   ★c. $x^2 - \frac{2}{3}x + \_\_\_\_\_$

5. The following scenarios guide you through the method of solving a quadratic equation by Completing the Square. First, move the constant term, c, to the other side of the equation.

   a. $x^2 - 10x + 21 = 0$

   b. $x^2 + 8x - 3 = 0$

6. Make the incomplete trinomial a perfect square trinomial by adding a constant term. To keep the equation balanced, add the constant term to both sides.

a. $x^2 - 10x \qquad = -21$

b. $x^2 + 8x \qquad = 3$

7. Factor the perfect square trinomial. Write it in factored form as $(x + \_\_)^2$ or $(x - \_\_)^2$.

a. $x^2 - 10x + 25 = 4$

b. $x^2 + 8x + 16 = 19$

8. Undo the exponent in $(x \pm \ \dots)^2$ to isolate the binomial inside the parentheses.

a. $(x - 5)^2 = 4$

b. $(x + 4)^2 = 19$

9. Isolate x to finish solving the equation.

a. $x - 5 = \pm 2$

b. $x + 4 = \pm\sqrt{19}$

10. Solve each equation using the method of Completing the Square.

a. $x^2 - 12x + 7 = 0$

b. $x^2 - 3x + 3 = 0$

11. The method of Completing the Square can only be applied if a trinomial has a leading coefficient of 1. Do not solve the following equations. Instead, describe the operation you need to perform to make it possible to solve the equation using the Completing the Square method.

a. $4x^2 - 16x + \_\_\_\_ = -7$

b. $-x^2 + 3x + \_\_\_\_ = -5$

SAMPLE PAGES

Section 3

Graphing Parabolas in Vertex Form

Use this page for taking notes or anything else that helps you learn.

Graphing Parabolas in Vertex Form

1. A **linear** function (its graph is a line) can be written in many forms. Each function below is in a different form but they all make the same line.

   $y = \frac{1}{2}x + 4$ $\quad$ $y - 3 = \frac{1}{2}(x + 2)$ $\quad$ $2x - 4y = -16$ $\quad$ $x = 2y - 8$

   Try to recall each form for a linear equation shown below.

   a. Slope-Intercept Form $\quad$ b. Point-Slope Form $\quad$ c. Standard Form

2. A **quadratic** function (its graph is a parabola) can also be written in many forms. The following functions all form the same parabola.

   $y = x^2 + 6x + 2$ $\quad$ $y + 7 = (x + 3)^2$ $\quad$ $y - x^2 = 6x + 2$ $\quad$ $y = (x + 3)^2 - 7$

   a. Do you remember the Standard Form for a parabola? Write it below.

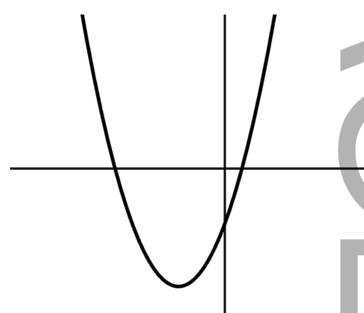
   b. Which equation above is in Standard Form?

3. When a quadratic function is in Standard Form, you can immediately know some characteristics of the parabola it forms. Consider the function $y = -2x^2 + 6x + 9$.

   a. Does this parabola look like a smile or a frown? How do you know?

   b. What are the coordinates of the y-intercept?

4. The shape of a parabola is symmetrical. A vertical line drawn through its vertex splits it in half. When a parabola's equation is in Standard Form ($y = ax^2 + bx + c$), how do you find the coordinates of its vertex?



5. Use $x = \frac{-b}{2a}$ to find the coordinates of the parabola's vertex.

   $y = 2x^2 + 12x - 3$

Though you can find a parabola's vertex when its equation is in Standard Form, you can also find the vertex when the equation is in a form called Vertex Form. You can use what you have learned about transformations to create this new form.

Applying Transformations to Parabolas

6. Suppose there is a function called $f(x)$. How does each transformation below move this function?

a. $f(x) - 5$ b. $f(x - 4)$ c. $f(x + 2) + 1$

7. Transformations are applied in a specific order, using the order of operations. How will each transformation shown change the function's x- and y-values? Write them in the order they occur.

a. $3f(x) + 2$ b. $-2f(x + 3)$ c. $f(4x) - 2$

8. Consider the function $f(x) = x^2$. Its vertex is located at $(0, 0)$. The notation below shows various transformations of this function. Each transformation moves the vertex of the parabola. Identify the new coordinates of the vertex for each function.

a. $f(x) + 2$ b. $f(x + 3)$ c. $f(x - 6) - 7$

9. Consider each function below. Think about the order in which each transformation would be applied and then identify the coordinates of each parabola's vertex.

a. $f(x) = x^2 + 5$ b. $f(x) = 2(x + 4)^2$ c. $f(x) = -3(x - 1)^2 + 5$

vertex: vertex: vertex:

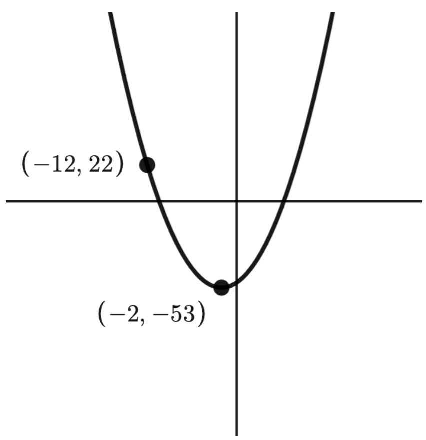
10. What are the coordinates of each parabola's vertex?

a. $f(x) = a(x - h)^2 + k$ b. $f(x) = a(x + h)^2 - k$

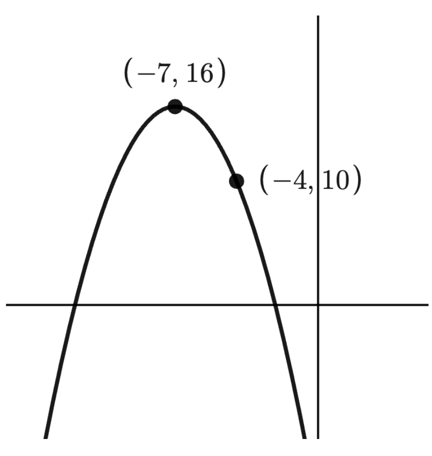
SAMPLE PAGES

21. If the "a" in $y = a(x - h)^2 + k$ is not easy to see in a graph, you can also find the a-value if you know the vertex and 1 more point on the parabola. For example, if the vertex of a parabola is $(3, -7)$, the parabola's equation is $y = a(x - 3)^2 - 7$. Suppose another point on the parabola is $(-2, 68)$. Use this point to find the a-value and write the complete equation.

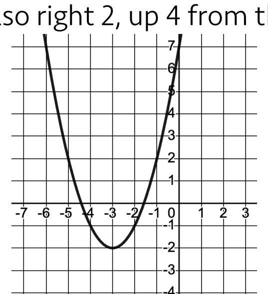
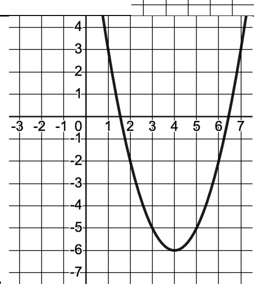
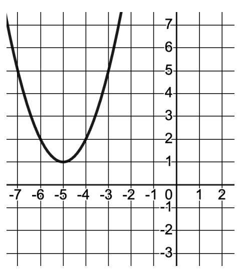
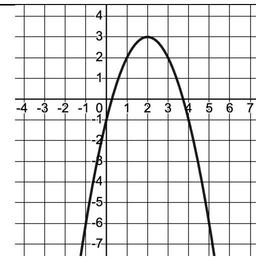
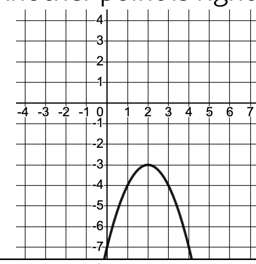
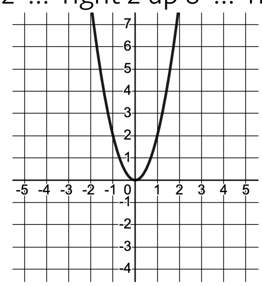
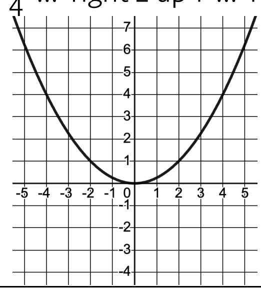
22. Write the equation of the parabola in Vertex Form.



23. Write the equation of the parabola in Vertex Form.



Answer Key

1.	a. $y = mx + b$ b. $y - y_1 = m(x - x_1)$ c. $Ax + By = C$
2.	a. $y = ax^2 + bx + c$ b. $y = x^2 + 6x + 2$
3.	a. frown; "a" is negative b. (0, 9)
4.	Use $x = \frac{-b}{2a}$ to find the x-value. Find y-value by putting the x-value into the function.
5.	vertex: (−3, −21) $x = \frac{-b}{2a} = \frac{-12}{2(2)} = -3$ $y = 2(-3)^2 + 12(-3) - 3 = -21$
6.	a. down 5 b. right 4 c. left 2, up 1
7.	a. multiply y by 3, then shift y up 2 b. shift x left 3, then multiply y by −2 c. multiply x by $\frac{1}{4}$, then shift y down 2
8.	a. (0, 2) b. (−3, 0) c. (6, −7)
9.	a. shift (0, 0) up 2: (0, 5) b. shift (0, 0) left 4: (−4, 0) then, multiply the y by 2: (−4, 0) c. shift (0, 0) right 1: (1, 0) then, multiply the y by −3: (1, 0) then, move the y up 5: (1, 5)
10.	a. (h, k) b. (−h, −k)
11.	a. (−3, −2) b. (20, 7) c. (0, 13)
12.	Move the vertex left 3, down 2. From the vertex, a point is right 1, up 1. A point is also right 2, up 4 from the vertex. 
13.	 a.  b.
14.	a. The vertical movements are multiplied by −1. From the vertex (2, 3), a point is right 1, down 1. Another point is right 2, down 4.  b. The function is multiplied by −1, so its new equation is $h(x) = -(x - 2)^2 - 3$. From the vertex (2, −3), a point is right 1, down 1. Another point is right 2, down 4. 
15.	a. 3 b. up $\frac{1}{2}$ c. right 1, down 5
16.	a. Multiply vertical movements by 2 right 1, up 2 ... right 2 up 8 ... right 3 up 18  b. Multiply vertical movements by $\frac{1}{4}$ right 1, up $\frac{1}{4}$ ... right 2 up 1 ... right 3 up $\frac{9}{4}$ 
17.	a. multiply the y-value by 3; (2, 4) → (2, 12) b. multiply y-value by $-\frac{1}{4}$; (2, 4) → (2, −1)
18.	Both parabolas are shown on the same grid. The solid curve is $g(x) = 2(x - 3)^2 - 4$. From the vertex, a point is right 1, up 2. Another point is right 2, up 8.

Section 4

Quadratic Optimization Scenarios

Use this page for taking notes or anything else that helps you learn.

Quadratic Maximum Revenue Scenarios

You can use what you know about finding a parabola's vertex to analyze scenarios with quadratic relationships. One type of scenario involves revenue (the income received from selling something).

1. A business sells hats for \$20 each.

   a. After selling 10 hats, their revenue is ___________.

   b. After selling H hats, their revenue is ___________.

   c. After selling $(10 + 2x)$ hats, their revenue is ___________.

2. If a business sells $(40 - 3x)$ items one month and each item costs $(6 + x)$ dollars, write the expression that represents their total revenue that month.

3. Since revenue is calculated by multiplying "cost per item" and "items sold," a revenue function can be written as a product of 2 factors: $R =$ (cost per item)(items sold). Consider the revenue function shown. Multiply the binomials to write the function in Standard Form.

   $$R = (3 + x)(20 - 2x)$$

4. A company's daily revenue changes depending on the price they charge for an item and the number of items they sell that day. If their revenue is modeled by the function shown, what is the maximum revenue they can achieve in a day?

   $$R = (6 + 0.5x)(80 - 4x)$$

Section 5

Quadratic Inequalities

Use this page for taking notes or anything else that helps you learn.

Solving Quadratic Inequalities by Graphing

1. After learning methods for solving quadratic equations, you can learn about quadratic inequalities.

   a. Consider the statement $x^2 = 4$. What x-values make this true?

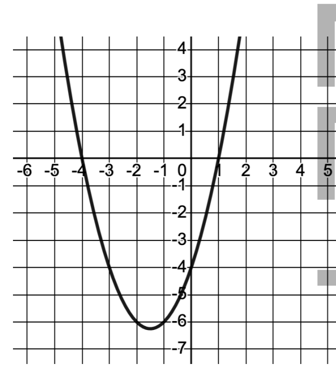
   b. Now consider the statement $x^2 < 4$. What numbers are less than 4 after they are squared?

   c. What numbers satisfy the inequality $x^2 \geq 9$?

2. Solve each inequality. Write the solution using interval notation.

   a. $x^2 \geq 0$ b. $x^2 > 0$ c. $x^2 \leq 16$ d. $x^2 > 25$

3. Use the graph of $f(x) = x^2 + 3x - 4$ to solve each equation or inequality below. Use interval notation.

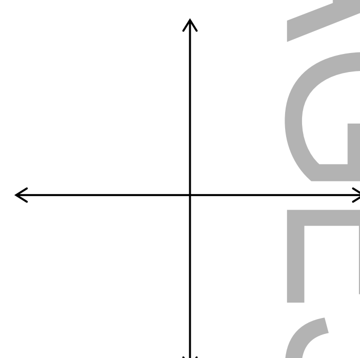


   a. For what x-values does $f(x) = 0$?

   b. For what x-values is $f(x) < 0$?

   c. For what x-values is $f(x) \leq -4$?

4. Now consider the inequality $x^2 - 2x - 8 < 0$.



   a. Before you find x-values that make the trinomial <u>less than</u> 0, first identify the values that make it <u>equal</u> 0. What are they?

   b. Now picture the shape of the graph of $y = x^2 - 2x - 8$. Draw a simple sketch and mark the x-intercepts. Using the graph, for which x-values is $y < 0$?

   c. Over what interval is $x^2 - 2x - 8 < 0$?

SAMPLE PAGES

CONTENTS

Unit 4

Circles & Nonlinear Systems

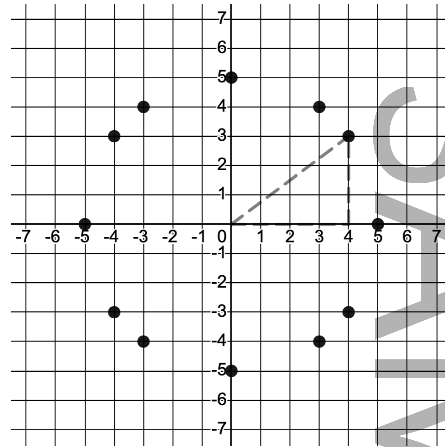
Section 1

Circles & Ellipses

Use this page for taking notes or anything else that helps you learn.

The Equation of a Circle

1. In a previous lesson, you may have learned that a circle's equation is $x^2 + y^2 = r^2$. To see why this equation produces a circle, look at the dots in the graph.



a. Each dot on the x- and y-axis is _____ units away from the origin, (0, 0).

b. The dashed lines in the graph form a right triangle. How long is its hypotenuse?

c. You can use the Pythagorean Theorem to show each point in the graph is ____ units away from the origin: (0, 0). For each of these points, $a^2 + b^2 =$ ____.

d. If you plot all points that are 5 units away from the origin, they form a circle. Since the circle's radius is 5, for any point on this circle, (x, y), it is true that $x^2 + y^2 =$ ____.

2. What is a circle's equation if its center is at (0, 0) and its radius is 6?

3. Write the Standard Form for a circle's equation with a center at (0, 0) and a radius of r.

Applying Transformations to Circles

A circle is not a function, but you can use transformations to change a circle's shape and center.

4. The circle $x^2 + y^2 = r^2$ can be moved by transformations. In what direction(s) does each transformation below move the circle?

a. $(x + 1)^2 + y^2 = r^2$ b. $x^2 + (y - 1)^2 = r^2$ c. $(x - 2)^2 + (y + 3)^2 = r^2$

5. The equation $x^2 + y^2 = 9$ makes a circle with a radius of 3 and a center at (0, 0). Use what you know about transformations to identify the coordinates of the center for each circle shown.

a. $(x + 3)^2 + y^2 = 4$ b. $x^2 + (y - 7)^2 = 25$ c. $(x - 5)^2 + (y + 2)^2 = 1$

Transforming a Circle to an Ellipse

23. Consider the circle formed by the equation $x^2 + y^2 = 1$. Change the equation as described.

a. Replace x with $\frac{1}{2}x$.

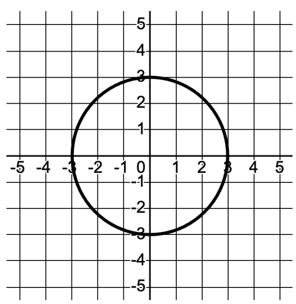
b. Replace y with $4y$.

24. Consider the circle formed by the equation $x^2 + y^2 = 1$. How does the circle's shape change if…

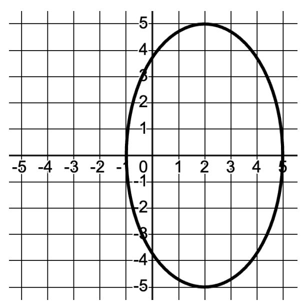
a. … x is replaced with $\frac{1}{3}x$?

b. … y is replaced with $5y$?

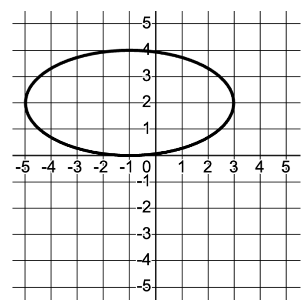
A stretched circle is called an **ellipse**. While a circle has a constant radius in all directions, an ellipse has different radii, so can be defined by its horizontal and vertical radius. Consider 3 examples.



center: (0, 0)
horizontal radius: 3
vertical radius: 3



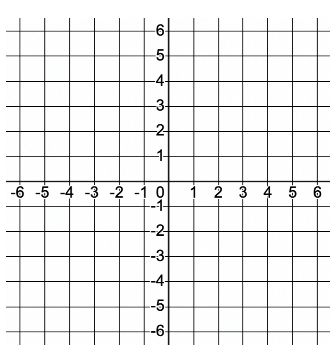
center: (2, 0)
horizontal radius: 3
vertical radius: 5



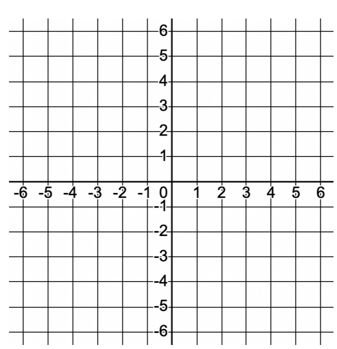
center: (−1, 2)
horizontal radius: 4
vertical radius: 2

25. Draw each ellipse described.

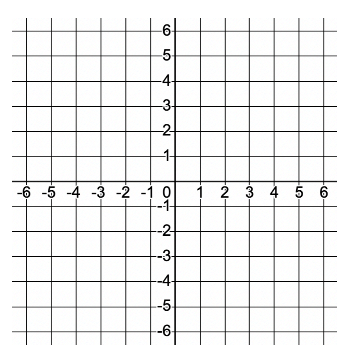
a. center: (0, 0)
horizontal radius: 2
vertical radius: 4



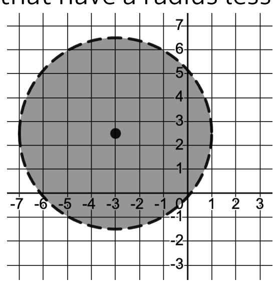
b. center: (0, 2)
horizontal radius: 1
vertical radius: 3

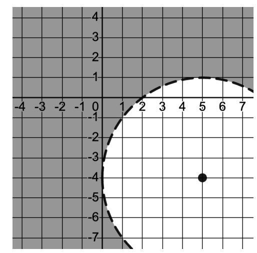


c. center: (1, −3)
horizontal radius: 5
vertical radius: 2



Answer Key

1.	a. 5 b. 5; solve $3^2 + 4^2 = c^2$ c. 5; $a^2 + b^2 = 5^2 = 25$ d. $x^2 + y^2 = 25$
2.	$x^2 + y^2 = 6^2 \rightarrow x^2 + y^2 = 36$
3.	$x^2 + y^2 = r^2$
4.	a. left 1 b. up 1 c. right 2, down 3
5.	a. $(-3, 0)$ b. $(0, 7)$ c. $(5, -2)$
6.	a. $\sqrt{25} \rightarrow 5$ b. $\sqrt{1} \rightarrow 1$ c. $\sqrt{2}$ d. $\sqrt{12} \rightarrow 2\sqrt{3}$
7.	a. (h, k) b. $(-h, -k)$
8.	a. $(x-2)^2 + (y-1)^2 = 16$ b. $(x+2)^2 + (y-3)^2 = 36$ c. $(x-7)^2 + (y+9)^2 = 144$
9.	a. center: $(3, 0)$ radius: 3 b. center: $(-2, 4)$ radius: 4
10.	a. center: $(0, -4)$ radius: 2 b. center: $(5, -2)$ radius: $\sqrt{6} \rightarrow \approx 2.5$
11.	a. d: $[-2, 2]$ b. d: $\left[5-\sqrt{6}, 5+\sqrt{6}\right]$
12.	center: $(-3, 2.5)$ radius: 4 Shade inside a dashed circle to show the points that have a radius less than 4. 
13.	Multiply both sides of the equation by 100 $(x+7)^2 + (y+11)^2 = 100$ center: $(-7, -11)$ radius: 10
14.	$x^2 + 8x + 16 + y^2 - 2y + 1 = 20$ $x^2 + 8x + y^2 - 2y = 3$
15.	a. $x^2 - 6x + \mathbf{9} + y^2 - 10y + \mathbf{25} = 2 + \mathbf{9} + \mathbf{25}$ b. $(x-3)^2 + (y-5)^2 = 36$ c. center: $(3, 5)$ radius: 6
16.	a. $x^2 - 8x + \mathbf{16} + y^2 + 12y + \mathbf{36} = 12 + \mathbf{52}$ $(x-4)^2 + (y+6)^2 = 64$ b. $x^2 + 14x + \mathbf{49} + y^2 - 2y + \mathbf{1} = -30 + \mathbf{50}$ $(x+7)^2 + (y-1)^2 = 20$
17.	$x^2 - 5x + \mathbf{\frac{25}{4}} + y^2 + 6y + \mathbf{9} = -3 + \mathbf{\frac{25}{4}} + \mathbf{9}$ $\left(x - \frac{5}{2}\right)^2 + (y+3)^2 = 6 + \mathbf{\frac{25}{4}}$ $\left(x - \frac{5}{2}\right)^2 + (y+3)^2 = \frac{49}{4}$

18.	center: $(2.5, -3)$ radius: $\frac{7}{2} \rightarrow 3.5$ domain: $[2.5 - 3.5, 2.5 + 3.5] \rightarrow [-1, 6]$ range: $[-3 - 3.5, -3 + 3.5] \rightarrow [-6.5, 0.5]$
19.	$x^2 - 10x + \mathbf{25} + y^2 + 8y + \mathbf{16} > -16 + \mathbf{41}$ $(x-5)^2 + (y+4)^2 > 25$ center: $(5, -4)$ radius: 5 Shade outside a dashed circle to show the points that have a radius more than 5. 
20.	Divide both sides by 2 $x^2 - 2x + y^2 + 6y - 8 = 0$ $x^2 - 2x + \mathbf{1} + y^2 + 6y + \mathbf{9} = 8 + \mathbf{10}$ $(x-1)^2 + (y+3)^2 = 18$ center: $(1, -3)$ radius: $\sqrt{18} \rightarrow 3\sqrt{2}$
21.	domain: $\left[1 - 3\sqrt{2}, 1 + 3\sqrt{2}\right]$ range: $\left[-3 - 3\sqrt{2}, -3 + 3\sqrt{2}\right]$
22.	The circle's diameter is the distance between the points. $d = \sqrt{(3+17)^2 + (-2-8)^2}$ $= \sqrt{500} = 10\sqrt{5} \rightarrow$ The radius is $5\sqrt{5}$. Area $= \pi r^2 = \pi\left(5\sqrt{5}\right)^2 = 125\pi$ units^2
23.	a. $\left(\frac{1}{2}x\right)^2 + y^2 = r^2 \rightarrow \frac{1}{4}x^2 + y^2 = r^2$ b. $x^2 + (4y)^2 = r^2 \rightarrow x^2 + 16y^2 = r^2$
24.	a. The x-values are multiplied by 3, so the circle is horizontally stretched and becomes 3 times wider. The circle becomes an ellipse. b. The y-values are multiplied by $\frac{1}{5}$, so the circle is vertically compressed and becomes $\frac{1}{5}$ of its original height. The circle becomes an ellipse.

Section 2

Nonlinear Systems

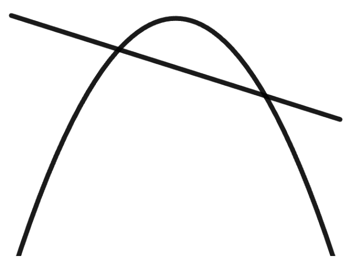
Use this page for taking notes or anything else that helps you learn.

Solving Nonlinear Systems Algebraically

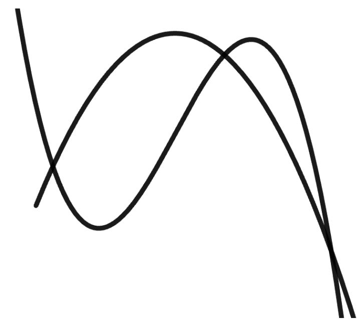
In the next scenarios, you will see systems of nonlinear equations. You can find the solutions to these systems by using the familiar methods of substitution and elimination.

1. Consider each system shown below. How many solutions does each system have?

a.



b.



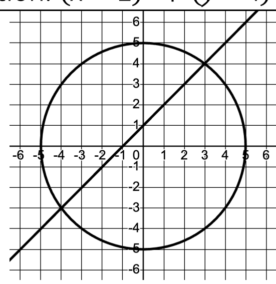
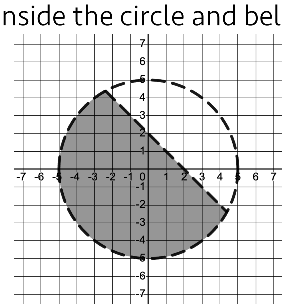
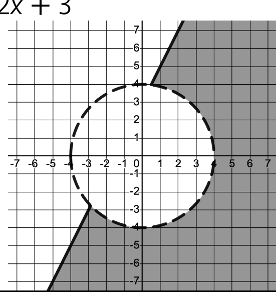
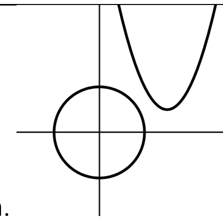
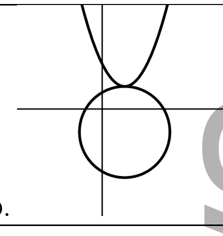
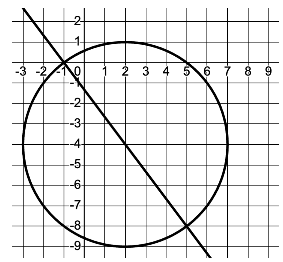
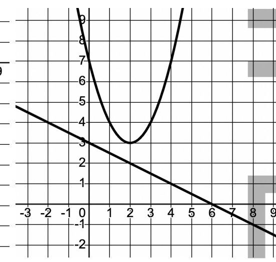
2. The solution to a system of 2 equations are the points where the graphs intersect. At these points, the 2 equations have equal x- and y-values. Consider the two functions below. Their graphs will intersect when their y-values are equal, or when $-x^2 - x + 5 = x^2 - x - 3$. Use the solution to this equation to find the solution to the system.

$y = -x^2 - x + 5$
$y = x^2 - x - 3$

3. Where do the two functions intersect? Determine the exact coordinates.

$y = (x - 1)^2 - 9$
$3 = 2x - y$

Answer Key

1.	a. 2 solutions (intersection points) b. 3
2.	Solve: $-x^2 - x + 5 = x^2 - x - 3$ $\rightarrow x = 2, -2$ Insert each x-value into either equation to find the y-value for that x-value. solutions: $(2, -1)$ and $(-2, 3)$
3.	$y = 2x - 3 \ \rightarrow \ 2x - 3 = (x-1)^2 - 9$ $\rightarrow \ 2x - 3 = x^2 - 2x + 1 - 9$ $\rightarrow \ 0 = x^2 - 4x - 5 \ \rightarrow \ x = 5, -1$ $x = 5 \ \rightarrow \ y = 7 \qquad x = -1 \ \rightarrow \ y = -5$ solutions: $(-1, -5)$ and $(5, 7)$
4.	The circle's center is the midpoint of the 2 solutions: $\left(\frac{-1+5}{2}, \frac{-5+7}{2}\right) \ \rightarrow \ (2, 1)$ The radius is the distance between $(2, 1)$ and either of the 2 solutions: $\sqrt{(5-2)^2 + (7-1)^2} \ \rightarrow \ \sqrt{9+36} \ \rightarrow \ 3\sqrt{5}$ equation: $(x-2)^2 + (y-1)^2 = 45$
5.	 solution: $(-4, -3)$ and $(3, 4)$ $x^2 + (x+1)^2 = 25$ $\rightarrow \ x^2 + \left(x^2 + 2x + 1\right) = 25$ $\rightarrow \ 2x^2 + 2x - 24 = 0 \ \rightarrow \ 2\left(x^2 + x - 12\right) = 0$ $\rightarrow \ 2(x+4)(x-3) = 0 \rightarrow \ x = -4, 3$
6.	a. shade inside the circle and below the line  b. $\begin{cases} x^2 + y^2 > 16 \\ y \leq 2x + 3 \end{cases}$ outside circle, below line 
7.	  a. b.
8.	$x^2 + \left(x^2 - 3\right)^2 = 9 \ \rightarrow \ x^2 + x^4 - 6x^2 + 9 = 9$ $\rightarrow \ x^4 - 5x^2 = 0 \ \rightarrow \ x^2\left(x^2 - 5\right) = 0$ $\rightarrow \ x = 0, \ \pm\sqrt{5}$ solution: $(0, -3)$, $\left(\sqrt{5}, 2\right)$, $\left(-\sqrt{5}, 2\right)$
9.	No solution; the system is 2 circles with the same center, $(0, 0)$, and different radii.
10.	a.  b.  solution:
11.	$-5t^2 + 18t + 10 = 13 + 2t$ $0 = 5t^2 - 16t + 3 \ \rightarrow \ 0 = (5t-1)(t-3)$ $\rightarrow \ t = \frac{1}{5}$ and 3 seconds The recording starts after $\frac{1}{5}$ sec and stops after 3 seconds so the length is $2\frac{4}{5}$ seconds.
12.	a. $7y = 3x + 29 \ \rightarrow \ y = \frac{3}{7}x + \frac{29}{7}$ $\rightarrow \ x^2 + \left(\frac{3}{7}x + \frac{29}{7}\right)^2 = 29$ $\rightarrow \ x^2 + \frac{9}{49}x^2 + \frac{174}{49}x + \frac{841}{49} = 29$ $\rightarrow \ 49x^2 + 9x^2 + 174x + 841 = 1421$ $\rightarrow \ 58x^2 + 174x - 580 = 0$ $\rightarrow \ 58\left(x^2 + 3x - 10\right) = 0$ $\rightarrow \ 58(x+5)(x-2) = 0$ $x = -5$ or $2 \ \rightarrow$ 5 miles to the west b. The radius of the circle is $\sqrt{29}$. The area is $\pi\left(\sqrt{29}\right)^2 \ \rightarrow \ 29\pi \approx 91$ mi^2. c. intersection points: $(-5, 2)$ and $(2, 5)$ distance: $\sqrt{7^2 + 3^2} \ \rightarrow \sqrt{58} \approx 7.6$ miles