SUMMIT MATH

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ALGEBRA 1

second edition

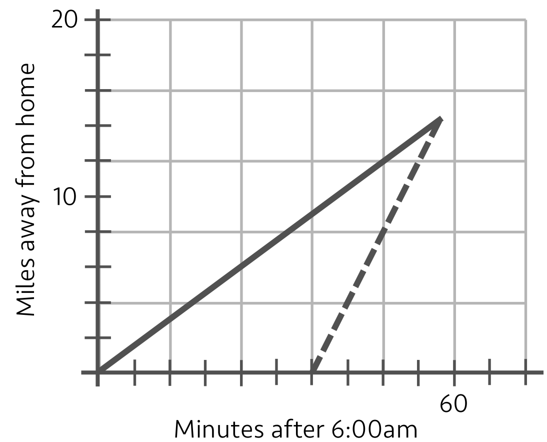
6 SYSTEMS OF LINEAR EQUATIONS & INEQUALITIES

CONTENTS

Section 1
INTRODUCTION TO INTERSECTING LINES

1. A boy gets up early one morning to go for a long ride on his bicycle. Thirty-six minutes later, his older sister realizes he left the inhaler that he uses for his asthma, so she gets in the car and tries to catch him.

Use the graph to estimate what time it is when the older sister catches her brother.



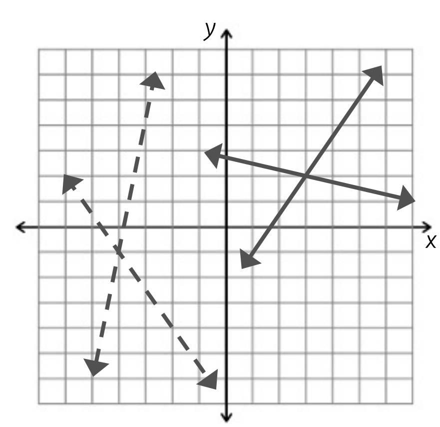
Looking at the graph, it seems impossible to determine the exact time at which the sister catches up with her brother because the grid lines (which help you determine exact numbers) do not line up nicely at the point where the dashed line intersects the path of the solid line.

When you first started graphing points on a Cartesian plane, you learned that there is information contained in each point. Similarly, since lines can be used to represent meaningful values, the intersection of two lines can also contain information.

2. Without using specific numbers, describe the information that is contained in the intersection point of the two lines in the scenario above.

In one sense, because you can see the point of intersection, the information that is contained in that point is directly in front of you. For now, though, you can probably only *estimate* where the two lines intersect. In the scenarios that follow, you will gradually discover how to calculate the exact coordinates of the point where two lines intersect.

3. You will eventually learn how to find the exact time that the sister catches her brother, but first, consider a simpler example. Looking at the graph shown, where do the two solid lines intersect?



4. Where do the two dashed lines intersect?

Section 3

THE SUBSTITUTION METHOD

You are now familiar with one method for determining the intersection point of two lines. The next method that you learn will use what you have learned in previous lessons about replacing a variable with another value.

33. Start with the equation $y = 2x + 7$.

a. What is the value of y if x is replaced with 5?

b. Rewrite the equation if $x = 10$.

c. Rewrite the equation if $x = 2M$.

d. Rewrite the equation if x is replaced with "$f + 1$".

e. Rewrite the equation if x is replaced with "$5 - 3y$".

34. Now start with the equation $2x - 4y = 12$.

a. Rewrite the equation if x is replaced with "$y + 2$".

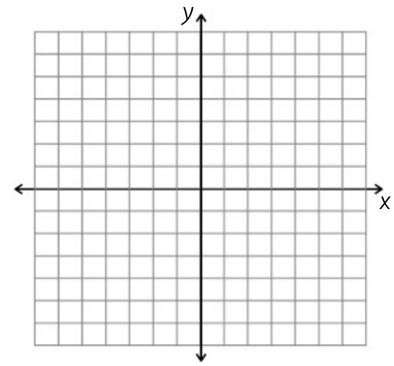
b. Rewrite the equation if $x = 3y - 2$.

c. Rewrite the equation if y is replaced with "$2x - 9$".

d. Rewrite the equation if $y = -4x - 3$".

35. In the equation $2x - 4y = 12$, there are two variables, x and y. In the previous scenario, when one of the variables is replaced with another expression, the resulting equation contains only one variable. Solve each of these resulting equations.

36. Consider Line 1, $y = -2x + 3$, and Line 2, $-6x + 2y = -14$. Graph them in the Cartesian plane provided.



Notice the point where the two lines intersect. At this point, the y-values are the same. In other words, the y-value for $y = -2x + 3$ is the same as the y-value for $-6x + 2y = -14$. Since the y-values are the same, and y equals $-2x + 3$, then the y-value in $-6x + 2y = -14$ also equals $-2x + 3$.

37. In the equation $-6x + 2y = -14$, replace the y with $-2x + 3$. An equation with two variables now contains only one variable. Solve this equation.

38. Your calculations in the previous scenario reveal that the lines intersect when $x = 2$. Since both lines pass through that intersection point, they will both have the same y-value when you replace x with 2 in each equation. Show this by replacing x with 2 in each equation and solving for y both times.

39. The method you just used is called the Substitution Method because you use an expression ($-2x + 3$) as a substitute for another variable (y). This method shows that the line $-6x + 2y = -14$ and the line $y = -2x + 3$ intersect at the point (___, ___).

40. Consider another pair of equations: $2x+3y=-1$ and $x = 5y + 6$. In the equation $2x+3y=-1$, replace the x-value with the other expression that x is "equal to" ($5y + 6$). Now that the equation only contains the variable y, solve for y.

41. Finish the work you started in the previous scenario to find the intersection point of the lines $2x+3y=-1$ and $x = 5y + 6$.

Section 4
THE ELIMINATION METHOD

Now that you have gained familiarity with the Substitution Method, you will learn about one more method for finding the intersection point of two lines. This next method combines basic arithmetic (addition, subtraction, etc...) with a clever bit of logic.

50. Consider the following scenarios. In each scenario, you are given two complete equations. Use those equations to determine the missing information in the incomplete third equation.

a. If $6x = A$ and $7x = B$, then $13x =$ ________.

b. If $10y = A$ and $-3y = B$, then ______ $= A + B$.

c. If $A = 2x + 3y$ and $B = 5x - y$, then $A + B =$ ________.

d. If $x - 2y = 100$ and $7x + 2y = 200$, then ________ $= 300$.

51. Fill in the missing information in the third equation.

a. If $5x + 3y = 12$ and $2x + 6y = 7$, then $7x + 9y =$ ______.

b. If $-3x + 2y = 9$ and $5x - 6y = -4$, then $2x - 4y =$ ______.

c. If $3x + 4y = 1$ and $5x + y = 4$, then $13x + 6y =$ ______.

52. Fill in the missing expression to make each addition scenario true.

a. $\begin{array}{r} 2x \\ +\ \underline{\quad\quad} \\ 7x \end{array}$ $\quad$ $\begin{array}{r} 5y \\ +\ \underline{\quad\quad} \\ 8y \end{array}$

b. $\begin{array}{r} 3x+2y=7 \\ +\ \underline{\quad\quad\quad\quad} \\ 8x+0y=3 \end{array}$

c. $\begin{array}{r} -2x+6y=10 \\ +\ \underline{\quad\quad\quad\quad} \\ x+0y=13 \end{array}$

53. Fill in the missing equation to make each subtraction scenario true.

a. $\begin{array}{r} 7x+8y=7 \\ -\ \underline{\quad\quad\quad\quad} \\ 2x+5y=3 \end{array}$

b. $\begin{array}{r} 4x+y=9 \\ -\ \underline{\quad\quad\quad\quad} \\ 0x+2y=3 \end{array}$

c. $\begin{array}{r} -6x+4y=5 \\ -\ \underline{\quad\quad\quad\quad} \\ x+0y=8 \end{array}$

54. Since subtracting can be challenging at times, scenarios will focus only on adding equations for now. Combine each pair of equations by adding downward. This will create a new equation.

a. $\begin{array}{r} -2x+4y=15 \\ +\quad \underline{2x+6y=5} \end{array}$

b. $\begin{array}{r} x+3y=60 \\ +\quad \underline{7x-3y=20} \end{array}$

c. $\begin{array}{r} 5x+2y=12 \\ +\quad \underline{-5x-3y=-9} \end{array}$

Section 6

SCENARIOS THAT INVOLVE SYSTEMS OF EQUATIONS

Each scenario so far has involved the intersection point of two lines. Since a line is a collection of points, with each point containing 2 pieces of information (an *x*-value and *y*-value), the methods for finding these *x*- and *y*-values can be used for any scenario that involves two related quantities. Use what you have learned so far to work through each of the following scenarios.

You have solved systems equations containing only *x*'s and *y*'s. If these variables are changed to other letters, it may seem confusing at first, but it does not change anything about what you have learned.

89. Use the **substitution** method to find the values of *w* and *E* that make both equations true.

$$E = \frac{1}{2}w - 7$$
$$4w + 2E = 26$$

90. Use the **substitution** method to find the values of *m* and *K* that make both equations true.

$$K = 150 - 12m$$
$$K = 130 - 9.5m$$

91. Use the **elimination** method to find the values of *f* and *g* that make both equations true.

$$1.25f + 5.25g = 10.50$$
$$f + g = 18$$

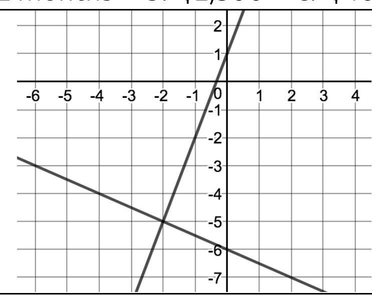
92. On May 1, 2014, the toll rate for passing through the Holland Tunnel into New York City was $13 for a car and $22 for a bus. Suppose a total of 3,000 cars and buses passed through the tunnel in one hour, and a total of $46,290 in tolls was collected.

a. Write an equation that relates the number of cars, *c*, and the number of buses, *b*, to the total number of cars and buses that passed through the tunnel in one hour.

b. Write an equation that shows the relationship between *c* and *b* and the total amount of money collected in tolls.

c. How many cars and how many buses passed through the tunnel during that hour?

Section 10
ANSWER KEY

1.	About 7:00am
2.	The intersection point shows two things: 1) how many miles the kids have ridden, and 2) what time it is when the sister catches her brother
3.	(3, 2)
4.	(−4, −1)
5.	a. 12 months b. $2,500 c. $40,000
6.	
7.	$y = mx + b$
8.	a. $y=\frac{2}{5}x+2$ b. $y=-\frac{3}{2}x$
9.	a. $y = 3$ b. $x = -4$
10.	a. 1 b. 3 c. 5 d. $\frac{1}{2}$
11.	a. 3 b. −16 c. $2\frac{1}{8}$ or $\frac{17}{8}$ or 2.125
12.	$y=-\frac{5}{2}$ or -2.5
13.	$y = -2$
14.	a. $y = -3x + 7$ b. $y = 3x - 7$
15.	$y=\frac{4}{5}x-2$
16.	a. negative b. neither c. positive
17.	Answer revealed later...
18.	Line 1: $y = -x + 1$ Line 2: $y = 2x + 5$
19.	a. 0 b. 7 c. Line 1: $y = 3$; Line 2: $y = 1$ d. Line 1: $x = 2$; Line 2: $x = -3$
20.	1
21.	2
22.	$x=-\frac{4}{3}$

23.	a. $y_1=\frac{7}{3}$ b. $y_2=\frac{7}{3}$
24.	$\left(-\frac{4}{3},\frac{7}{3}\right)$ or $\left(-1\frac{1}{3},2\frac{1}{3}\right)$
25.	(2, −3) Make the equations equal each other. Solve: $2x - 7 = -4x + 5$ The equations are equal when $x = 2$. Find y by letting $x = 2$ in either equation.
26.	a. (3, −5) Solve: $x-8=-5x+10$ b. $\left(\frac{5}{2},\frac{7}{2}\right)$ or (2.5, 3.5) Solve: $\frac{1}{5}x+3=-3x+11$
27.	(18, −7) Solve: $\frac{1}{3}x-13=-\frac{5}{3}x+23$
28.	They intersect at (3, −2). Write equations in Slope-Intercept Form as $y=3x-11$ and $y=-4x+10$.
29.	They intersect at (4, 2). Write equations in Slope-Intercept Form as $y=\frac{3}{2}x-4$ and $y=-\frac{3}{4}x+5$.
30.	$y=\frac{1}{2}x+13$
31.	$\left(\frac{8}{3},\frac{40}{3}\right)$
32.	brother: $y=\frac{1}{4}x$ sister: $y=\frac{2}{3}x-24$ $x=\frac{288}{5}$ minutes (57.6 minutes) At 6:57.6am (approx. 6:58am)
33.	a. $y=2(5)+7$ b. $y=2(10)+7$ c. $y=2(2M)+7$ d. $y=2(f+1)+7$ e. $y=2(5-3y)+7$
34.	a. $2(y+2)-4y=12$ b. $2(3y-2)-4y=12$ c. $2x-4(2x-9)=12$ d. $2x-4(-4x-3)=12$

