SUMMIT MATH

Learn at your **OWN** pace.

ADVANCED ALGEBRA & TRIGONOMETRY SERIES

BOOK 4

UNIT 7: **EXPONENTIAL** EQUATIONS & FUNCTIONS

UNIT 8: **LOGARITHMIC PROPERTIES,** EQUATIONS & FUNCTIONS

INTRODUCTION

Learning math through Guided Discovery:
Guided Discovery is designed to help you experience a sense of discovery as you learn each topic.

Why this curriculum series is named Summit Math:
Learning through Guided Discovery is like hiking. Learning and hiking both require effort and persistence and people naturally move at different paces, but they can reach the summit if they keep moving forward. Whether you race rapidly through the book or step slowly through each scenario, this textbook is designed to keep advancing your learning until you reach the summit.

Guided Discovery Scenarios:
The Guided Discovery Scenarios in this book are written and arranged to show you that new math concepts are related to previous concepts you already know. Try each scenario on your own first, check your answer when you finish, and then fix any mistakes, if needed. Making mistakes and struggling are essential parts of the learning process.

Homework & Extra Practice Scenarios:
After you complete the scenarios in each Guided Discovery section, extra practice will help you develop better memory of each topic. Use the Homework & Extra Practice Scenarios to improve your understanding and to increase your retention.

The Answer Key:
The Answer Key is included to give you teacher-like guidance. When you finish a scenario, you can get immediate feedback. If the Answer Key does not help you fully understand the scenario, try to get additional guidance from another student or a teacher.

Find more resources at:
www.summitmath.com

GUIDED DISCOVERY SCENARIOS

The Guided Discovery Scenarios are designed to help you experience a sense of discovery as you learn. Explanations are brief and carefully timed to allow you to build your learning at a comfortable pace. The Answer Key supports your learning by giving you immediate feedback and helping you understand each step before moving on.

As you complete each scenario, follow these steps.

Step 1: Try the scenario.
Read the scenario carefully and try to work through it. Be willing to struggle.

Step 2: Check the Answer Key.
The Answer Key can guide you through the scenario, show you new ideas, or help you find mistakes in your steps.

Step 3: Fix your mistakes, if needed.
Mistakes are learning opportunities. If your result does not match the Answer Key, check your work and look for errors. If you still need guidance, seek help from a classmate or teacher.

When you are ready, try the next scenario and repeat this cycle.

SAMPLE PAGES

CONTENTS

Unit 7

Exponential Functions & Equations

Section 1

Rational Exponents

Use this page for taking notes or anything else that helps you learn.

Introduction to Rational Exponents

This section will use what you know about exponents and roots to help you learn about **ratio**nal exponents, exponents that are fractions (ratios). First, review some exponent properties.

1. Consider the following expressions. Write each expression in the form A^B.

a. $x^4 \cdot x^4$ b. $2^3 \cdot 2^3$ c. $9^{1/2} \cdot 9^{1/2}$ d. $8^{1/3} \cdot 8^{1/3}$

2. Write each expression in the form A^B.

a. $(x^5)^2$ b. $\left(2^3\right)^4$ c. $\left(9^{1/2}\right)^5$ d. $\left(4^3\right)^{1/2}$

3. You know that 9^2 is 81, but what is the value of $9^{1/2}$? Type the expression $9^{0.5}$ into a calculator. A calculator will show you that $9^{1/2} =$ ____. To see why $9^{1/2}$ has that value, complete each part below.

a. Simplify $\left(9^{1/2}\right)^2$. b. Simplify 3^2. c. If $\left(9^{1/2}\right)^2 = 3^2$, then $9^{1/2} =$ ______.

4. If $\left(9^{1/2}\right)^2$ is 9, then $9^{1/2}$ is the square root of 9. An exponent of one-half and a "square root" symbol have the same meaning. Rewrite each expression using a square root symbol instead of an exponent. The first one is done for you.

a. $25^{1/2}$ b. $100^{1/2}$ c. $x^{1/2}$ d. $3^{1/2}$

$\sqrt{25}$

5. Notice that the expression $100^{1/2}$ is <u>not</u> one-half of 100. Evaluate each expression shown.

a. $36^{1/2}$ b. $49^{1/2}$ c. $64^{1/2}$ d. $\left(\frac{9}{25}\right)^{1/2}$

6. Since $x^{1/2} = \sqrt{x}$, then $27^{1/2} = \sqrt{27} = 3\sqrt{3}$. What is the simplified radical form of each expression?

a. $8^{1/2}$ b. $12^{1/2}$

7. What is the value of $8^{1/3}$? A calculator will show you that $8^{1/3} =$ ____. To see why $8^{1/3}$ has that value, complete each part below.

a. Simplify $\left(8^{1/3}\right)^3$.

b. Simplify 2^3.

c. If $\left(8^{1/3}\right)^3 = 2^3$, then $8^{1/3} =$______.

8. If $\left(8^{1/3}\right)^3$ is 8, then $8^{1/3}$ is the cube root of 8. An exponent of one-third and a "cube root" symbol have the same meaning. Rewrite each expression using a cube root symbol instead of an exponent. The first one is done for you.

a. $64^{1/3}$

$\sqrt[3]{64}$

b. $125^{1/3}$

c. $x^{1/3}$

d. $10^{1/3}$

9. Notice that the expression $27^{1/3}$ does not mean one-third of 27. Evaluate each expression shown.

a. $64^{1/3}$

b. $1000^{1/3}$

c. $1^{1/3}$

d. $\left(\frac{8}{27}\right)^{1/3}$

Now that you have learned that $x^{1/2} = \sqrt{x}$ and $x^{1/3} = \sqrt[3]{x}$, you can build on this and discover how to evaluate other rational exponents like $1/4$, $2/3$, $3/4$ and so on....

10. Consider the expression $8^{2/3}$. You can evaluate $8^{2/3}$ if you see it in a different way. First, simplify each expression below and write it in the form A^B.

a. $\left(x^{1/4}\right)^3$

b. $\left(27^{1/3}\right)^2$

c. $\left(25^{1/2}\right)^3$

11. In the previous scenario, you can use the Power Rule for exponents to show that $\left(25^{1/2}\right)^3 = 25^{3/2}$. If you reverse this property, $8^{2/3}$ is $\left(8^{1/3}\right)^2$. Rewrite each expression in the form $(x^A)^B$.

a. $8^{5/3} =$

b. $16^{3/2} =$

Answer Key

1.	a. $x^{4+4} \rightarrow x^{8}$ b. $2^{3+3} \rightarrow 2^{6}$ c. $9^{2/2} \rightarrow 9^{1}$ or 9 d. $8^{2/3}$
2.	a. $x^{5 \cdot 2} \rightarrow x^{10}$ b. 2^{12} c. $9^{5/2}$ d. $4^{3/2}$
3.	a. $9^{1} \rightarrow 9$ b. 9 c. $9^{1/2} = 3$
4.	b. $\sqrt{100}$ c. $\sqrt{x}$ d. $\sqrt{3}$
5.	a. 6 b. 7 c. 8 d. $\frac{3}{5}$
6.	a. $\sqrt{8} \rightarrow \sqrt{4}\sqrt{2} \rightarrow 2\sqrt{2}$ b. $\sqrt{12} \rightarrow \sqrt{4}\sqrt{3} \rightarrow 2\sqrt{3}$
7.	a. $8^{1} \rightarrow 8$ b. 8 c. $8^{1/3} = 2$
8.	b. $\sqrt[3]{125}$ c. $\sqrt[3]{x}$ d. $\sqrt[3]{10}$
9.	a. 4 b. 10 c. 1 d. $\frac{2}{3}$
10.	a. $x^{1/4 \cdot 3} \rightarrow x^{3/4}$ b. $27^{2/3}$ c. $25^{3/2}$
11.	a. $\left(8^{1/3}\right)^{5}$ b. $\left(16^{1/2}\right)^{3}$
12.	a. $100^{5/2}$ b. $\left(\sqrt{100}\right)^{5}$
13.	a. $\left(\sqrt[3]{8}\right)^{4}$ b. $\left(\sqrt{16}\right)^{3}$
14.	a. $\left(\sqrt[7]{x}\right)^{5}$ b. $\left(\sqrt[B]{x}\right)^{A}$
15.	a. $11^{2/7}$ b. $5^{9/4}$
16.	a. $2^{5/3}$ b. $9^{4/11}$
17.	a. $x^{A/B}$ b. $y^{B/A}$
18.	a. $(2)^{2} \rightarrow 4$ b. $\left(\sqrt[3]{8}\right)^{4} \rightarrow (2)^{4} \rightarrow 16$

19.	a. $\left(\sqrt{9}\right)^{3} \rightarrow (3)^{3} \rightarrow 27$ b. $\left(\sqrt[4]{16}\right)^{3} \rightarrow (2)^{3} \rightarrow 8$
20.	a. $\left(\sqrt[3]{27}\right)^{4} \rightarrow (3)^{4} \rightarrow 81$ b. $\left(\sqrt[5]{32}\right)^{3} \rightarrow (2)^{3} \rightarrow 8$
21.	a. $\left(\sqrt[3]{-27}\right)^{2} \rightarrow (-3)^{2} \rightarrow 9$ b. $\left(\sqrt{\frac{4}{9}}\right)^{3} \rightarrow \left(\frac{2}{3}\right)^{3} \rightarrow \frac{8}{27}$
22.	In Option #1, it is easier to find the cube root of 8 first.
23.	a. $32^{4/5} \cdot (x^{5})^{4/5} \rightarrow 2^{4} \cdot x^{4} \rightarrow 16x^{4}$ b. $(-64)^{2/3} \cdot \left(x^{12}\right)^{2/3} \rightarrow 16 \cdot x^{8} \rightarrow 16x^{8}$
24.	a. $\left(\frac{1}{8}\right)^{1/3} \rightarrow \sqrt[3]{\frac{1}{8}} \rightarrow \frac{1}{2}$ b. $4^{1/2} \rightarrow \sqrt{4} \rightarrow 2$
25.	a. $\left(\frac{1}{100}\right)^{3/2} \rightarrow \frac{1}{(100)^{3/2}} \rightarrow \frac{1}{1000}$ b. $27^{2/3} \rightarrow 9$
26.	a. $4^{7/2} \rightarrow \left(\sqrt{4}\right)^{7} \rightarrow 2^{7} \rightarrow 128$ b. $\left(\sqrt{-9}\right)^{3} \rightarrow (3i)^{3} \rightarrow 27i^{3} \rightarrow 27(-i) \rightarrow -27i$ c. $(10{,}000)^{3/4} \rightarrow \left(\sqrt[4]{10{,}000}\right)^{3}$ $\rightarrow (10)^{3} \rightarrow 1{,}000$

Section 2

Exponential Equations & Applications

Use this page for taking notes or anything else that helps you learn.

Solving Equations with Rational Exponents

1. To review what you have learned about exponents, try to simplify each expression.

a. $4^{-1} + 5^{-1}$ b. $\left(2^{3/5}\right)^{5/3}$ c. $\dfrac{2^{-1}}{5^{-2}}$ d. $27^{-2/3}$

2. Simplify each expression.

a. $\left(x^2\right)^{1/2}$ b. $\left(x^2\right)^{1/6}$ c. $\left(x^{1/3}\right)^3$ d. $\left(x^{3/2}\right)^{4/3}$ e. $\left(x^{2/3}\right)^{3/2}$

3. When you solve the equation $x^3 = 8$, you undo the exponent of 3 by finding the cube root of both sides. Another way to think about this is to undo an exponent with its reciprocal.

a. $x^3 = 8 \rightarrow \left(x^3\right)^{1/3} = (8)^{1/3} \rightarrow \sqrt[3]{x^3} = \sqrt[3]{8} \rightarrow x =$ _____

b. $x^5 = 100{,}000 \rightarrow (x^5)^{1/5} = (100{,}000)^{1/5} \rightarrow x = \sqrt[5]{100{,}000} \rightarrow x =$ _____

4. Solve each equation by raising each side to the reciprocal exponent.

a. $x^{1/2} = 3$ b. $x^{1/3} = 3$ c. $x^{1/4} = 3$

$\left(x^{1/2}\right)^2 = (3)^2$

5. Solve each equation by raising each side to the reciprocal exponent.

a. $x^{2/3} = 25$ b. $x^{4/3} = 16$ b. $(2x)^{4/5} = 16$

Answer Key

1.	a. $\frac{1}{4}+\frac{1}{5} \rightarrow \frac{5}{20}+\frac{4}{20} \rightarrow \frac{9}{20}$ b. $2^{\frac{3}{5}\cdot\frac{5}{3}} \rightarrow 2^1 \rightarrow 2$ c. $\frac{5^2}{2^1} \rightarrow \frac{25}{2}$ d. $\left(\frac{1}{27}\right)^{2/3} \rightarrow \frac{1}{27^{2/3}} \rightarrow \frac{1}{\left(\sqrt[3]{27}\right)^2} \rightarrow \frac{1}{9}$
2.	a. x b. $x^{1/3}$ c. x d. x^2 e. x
3.	a. $x=2$ b. $x=10$
4.	a. $x=9$ b. $\left(x^{1/3}\right)^3=(3)^3 \rightarrow x=27$ c. $\left(x^{1/4}\right)^4=(3)^4 \rightarrow x=81$
5.	a. $\left(x^{2/3}\right)^{3/2}=(25)^{3/2} \rightarrow x=5^3 \rightarrow x=125$ b. $\left(x^{4/3}\right)^{3/4}=(16)^{3/4} \rightarrow x=2^3 \rightarrow x=8$ c. $\left((2x)^{4/5}\right)^{5/4}=(16)^{5/4} \rightarrow 2x=32 \rightarrow x=16$
6.	a. $\left(x^{-2/3}\right)^{-3/2}=(4)^{-3/2} \rightarrow x=\frac{1}{4^{3/2}} \rightarrow \frac{1}{8}$ b. $\left(\left(\frac{x}{3}\right)^{-4/3}\right)^{-3/4}=(16)^{-3/4}$ $\rightarrow \frac{x}{3}=\frac{1}{8} \rightarrow x=\frac{3}{8}$
7.	a. $x+3=9^{3/2} \rightarrow x+3=27 \rightarrow x=24$ b. $(x-4)^{3/5}=8 \rightarrow x-4=8^{5/3}$ $\rightarrow x-4=32 \rightarrow x=36$
8.	$x=2$
9.	a. $2x=6 \rightarrow x=3$ b. $x+12=5x \rightarrow x=3$ c. $x-5=3x+1 \rightarrow x=-3$
10.	Make the bases match: rewrite 8 as 2^3. $2^{x+14}=\left(2^3\right)^x \rightarrow 2^{x+14}=2^{3x}$
11.	a. 2^{5x+10} b. $2^1\cdot 2^{2x} \rightarrow 2^{2x+1}$ c. $2^x\cdot\left(2^4\right)^{x-1} \rightarrow 2^x\cdot 2^{4x-4} \rightarrow 2^{5x-4}$ d. $2^3\cdot\left(2^2\right)^x \rightarrow 2^3\cdot 2^{2x} \rightarrow 2^{2x+3}$
12.	a. $\left(3^2\right)^{x-4}=3^{x+1} \rightarrow 3^{2x-8}=3^{x+1} \rightarrow x=9$ b. $\left(3^2\right)^{x+2}=\left(3^3\right)^{3x+1}$ $\rightarrow 3^{2x+4}=3^{9x+3} \rightarrow x=\frac{1}{7}$

13.	a. $\left(2^3\right)^{x-4}=\left(2^{1/2}\right)^{x+1} \rightarrow x=5$ b. $2^2\cdot\left(2^3\right)^{3x}=\left(2^{-4}\right)^{3-x} \rightarrow x=-2.8$
14.	a. $(u-2)(u-1)=0 \rightarrow u=2$ or 1 $2^x=2$ or $2^x=1 \rightarrow x=1$ or 0 b. $u=3^x \rightarrow u^2-2u+1=0$ $(u-1)(u-1)=0 \rightarrow u=1$ $3^x=1 \rightarrow x=0$ c. $u=4^x \rightarrow u^2-2u=0$ $u(u-2)=0 \rightarrow u=0$ or 2 $4^x=0 \rightarrow$ no solution $4^x=2 \rightarrow x=\frac{1}{2}$
15.	a. 100(1.8) → 180 people b. 100(1.8)(1.8) → $100(1.8)^2$ → 324 people c. $100(1.8)^4$ → 1,049.78 → 1,050 people
16.	a. 16 is 2 sets of 8 $600(0.80)^2$ → 384 mL b. 40 is 5 sets of 8 $600(0.80)^5$ → 196.608 mL c. $600(0.80)^{1/2}$ → 536.656 mL
17.	a. 1 year is 3 sets of 4 months $(1.25)^3$ → 1.9531 → 95.3% increase b. 1 month is one-fourth of 4 months $(1.25)^{1/4}$ → 1.0574 → 5.7% increase $(1.25)^{M/4}\times 100$
18.	8 days is 2 sets of 4 days 50% of 50% → 25%
19.	12 hours is 3 sets of 4 hours The original amount is halved 3 times. $400(0.50)^3$ → 50mg
20.	Solve: $20=40e^{-0.2t}$ Find the intersection of $y=20$ and $y=40e^{-0.2t}$ $t=3.466$ around 3.47 decades, or 34.7 years
21.	The compound is halved 2 times, so the half-life is 4 months.

Section 3

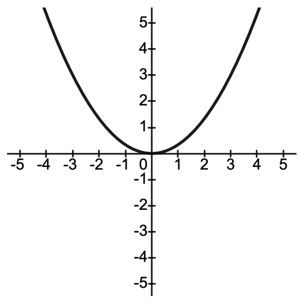
Exponential Functions: Part 1

Use this page for taking notes or anything else that helps you learn.

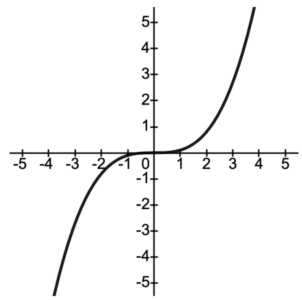
Exponential Patterns

1. Some functions increase as *x* increases in the positive direction. Three of these functions, with curved shapes, are shown. Try to recall the parent function for each graph (not the actual function).

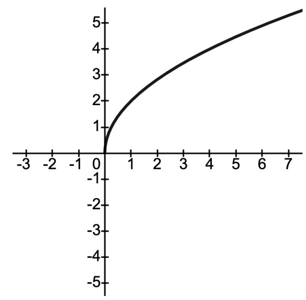
a.



b.



c.



2. In the next scenarios, you will learn about another function with a curved, increasing shape. Analyze each sequence below and find the pattern. What expression or number comes next in each sequence?

a. $2, \ 2 \cdot 2, \ 2 \cdot 2 \cdot 2, \ \ldots$

b. 3, 9, 27, . . .

3. Identify the next expression or number in each sequence.

a. $5, \ 5 \cdot 3, \ 5 \cdot 3 \cdot 3, \ \ldots$

b. 6, 12, 24, . . .

> The previous sequences are made by repeated multiplication, which can be represented with exponents. Consider the sequence 2, 4, 8, 16. Using exponents, the sequence is $2^1, 2^2, 2^3, 2^4$.

4. Use exponents to represent this sequence: 5, 25, 125.

> Consider a more complex sequence: 20, 40, 80, 160.
> Using exponents, this sequence is $10 \cdot 2^1, \ 10 \cdot 2^2, \ 10 \cdot 2^3, \ 10 \cdot 2^4$.

5. Use exponents to represent each sequence.

a. 6, 12, 24, 48, ...

b. 20, 200, 2000, ...

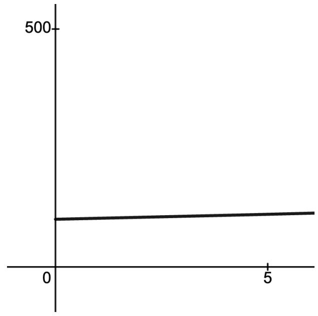
6. Use exponents to represent the 5th term in each sequence.

a. 6, 12, 24, 48, ...

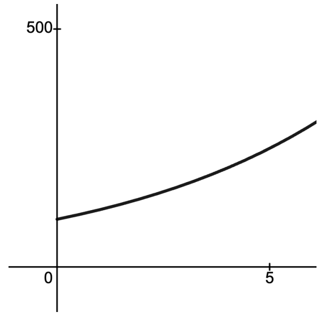
b. 20, 200, 2000, 20000, ...

Each function in the previous scenario is in the form $y = a(b)^x$. As b gets larger, the function grows more rapidly. You can see this in the graphs below.

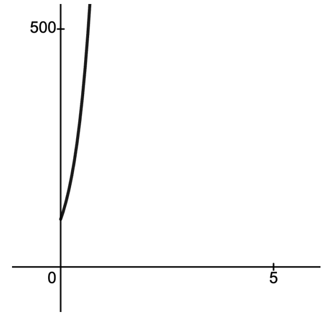
$y = 100(1.02)^x$



$y = 100(1.2)^x$



$y = 100(12)^x$

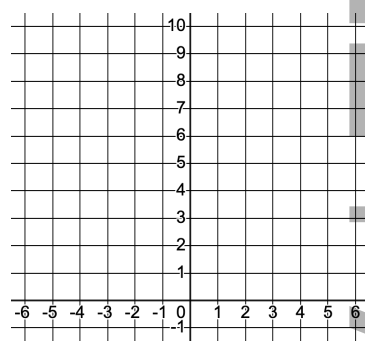


11. The graphs in the previous scenario do not show any negative x-values. Without a calculator, recall what you have learned about negative exponents and simplify each expression.

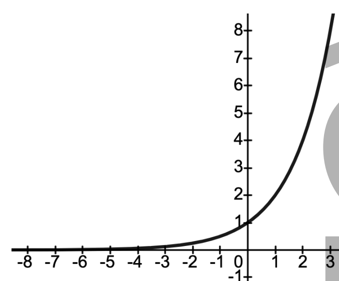
a. 4^{-1} b. 3^{-2} c. $\left(\frac{1}{2}\right)^{-3}$ d. $\left(\frac{3}{5}\right)^{-2}$

12. Graph the function $y = 2^x$. Find in the table and then plot those 6 points before you draw the curve.

x	-2	-1	0	1	2	3
y						



13. The graph of $y = 2^x$ is shown. On the right side of the y-axis, the y-values increase rapidly and go off the graph. On the left side of the y-axis, the y-values get so small they approach 0.



a. To help understand the left side behavior, simplify each expression below.

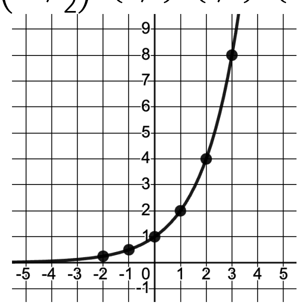
$2^{-3} =$ $2^{-6} =$

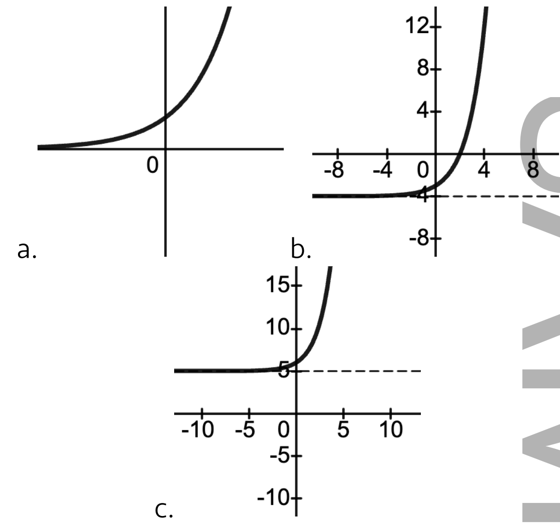
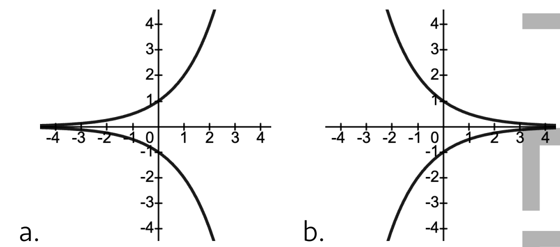
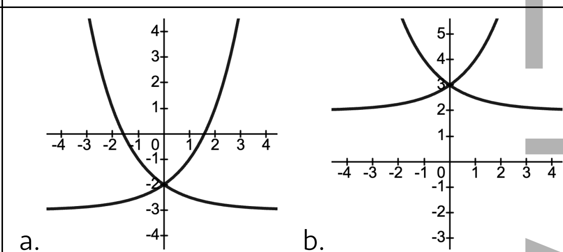
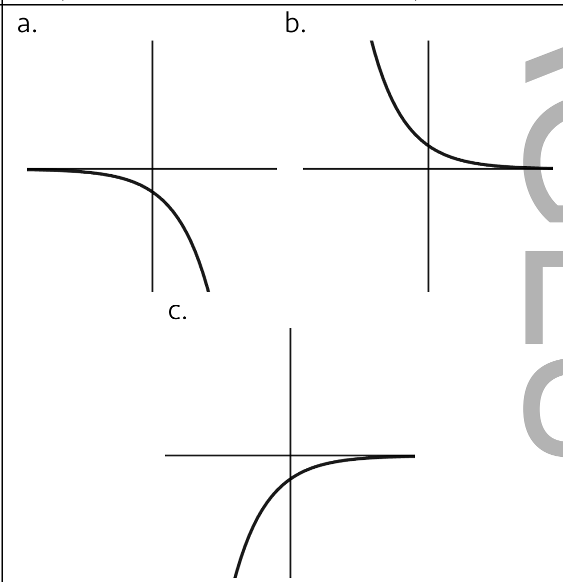
b. What is the end behavior of the function $y = 2^x$?

As $x \to \infty$, $y \to$ As $x \to -\infty$, $y \to$

c. For which x-values is 2^x negative?

Answer Key

1.	a. $y = x^2$ b. $y = x^3$ c. $y = \sqrt{x}$
2.	a. $2 \cdot 2 \cdot 2 \cdot 2 \rightarrow 16$ b. $27 \cdot 3 \rightarrow 81$
3.	a. $5 \cdot 3 \cdot 3 \cdot 3 \rightarrow 135$ b. $24 \cdot 2 \rightarrow 48$
4.	$5^1,\ 5^2,\ 5^3$
5.	a. $3 \cdot 2^1, 3 \cdot 2^2, 3 \cdot 2^3, 3 \cdot 2^4 \ldots$ b. $2 \cdot 10^1, 2 \cdot 10^2, 2 \cdot 10^3, 2 \cdot 10^4 \ldots$
6.	a. $3 \cdot 2^5$ b. $2 \cdot 10^5$
7.	a. $3 \cdot 2^n$ b. $2 \cdot 10^n$
8.	a. $y = 3^x$ b. $y = 3 \cdot 2^x$ c. $y = 2 \cdot 10^x$
9.	a. 1 b. -8 c. $3(100) \rightarrow 300$ d. $12\left(\frac{1}{4}\right) \rightarrow 3$
10.	$x = 0 \rightarrow 100,\quad x = 1 \rightarrow 102,\quad x = 2 \rightarrow 104$ $x = 0 \rightarrow 100,\quad x = 1 \rightarrow 120,\quad x = 2 \rightarrow 144$ $x = 0 \rightarrow 100,\quad x = 1 \rightarrow 1200,\quad x = 2 \rightarrow 14400$
11.	a. $\frac{1}{4}$ b. $\frac{1}{9}$ c. $2^3 \rightarrow 8$ d. $\left(\frac{5}{3}\right)^2 \rightarrow \frac{25}{9}$
12.	$\left(-2, \frac{1}{4}\right)$ $\left(-1, \frac{1}{2}\right)$ $(0, 1)$ $(1, 2)$ $(2, 4)$ $(3, 8)$ 
13.	a. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$ b. As $x \rightarrow \infty, y \rightarrow \infty$ As $x \rightarrow -\infty, y \rightarrow 0$ c. Though 2^x approaches 0, it never equals 0 and never becomes negative.
14.	a. $y = -3$ b. $y = 4$ c. $y = 2$
15.	a. $y = 0$ b. $y = -4$ c. $y = 5$ d. $y = 0$; the graph is shifted right 6

16.	
17.	
18.	
19.	a. y-axis b. x-axis c. x d. y e. both
20.	

Section 4

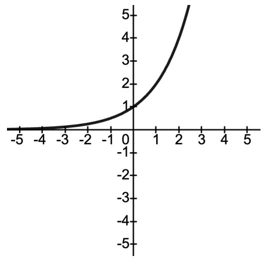
Exponential Functions: Part 2

Use this page for taking notes or anything else that helps you learn.

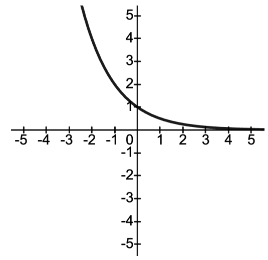
Exponential Growth & Decay

Exponential patterns are modeled by the function $y = a(b)^x$. When the function increases to the right, it is called exponential growth. If it decreases to the right, it is exponential decay.

Exponential Growth



Exponential Decay



1. Consider the function $y = 100(1.5)^x$. As x increases, will the y-values increase or decrease? Why?

2. Consider the function $y = 100(0.5)^x$. As x increases, will the y-values increase or decrease? Why?

3. In the function $y = a(b)^x$, what b-values cause exponential decay?

4. Label each function as growth or decay.

   a. $y = 6(0.98)^x$ b. $y = 0.4(1.02)^x$ c. $y = 9(2.3)^x$

5. Label each function as growth or decay.

   a. $y = 0.81(0.6 + 0.6)^x$ b. $y = 9(1 - 0.3)^x$ c. $y = 4.03(2 - 1.5)^x$

6. Sketch a simple graph of each function. Does it model exponential growth or decay?

   a. $y = 2^x$ b. $y = 2^{-x}$

SAMPLE PAGES

Percent Scenarios

An exponential function models repeated multiplication. The expression $a(b)^x$ means "a" is multiplied by "b" a total of x times. An exponential function can also represent percent scenarios.

15. In the expression 500(1.2), multiplying 500 by 1.2 increases 500 by ______%.

16. In the expression 30(0.9), multiplying 30 by 0.9 decreases 30 by ______%.

17. A video has been viewed 24 times and the views increase by 50% every hour. After 1 hour, it has 24(1.5) views. After 2 hours, it has 24(1.5)(1.5) or $24(1.5)^2$ views.

    a. After 3 hours, it has ___________ views.   b. After h hours, it has ___________ views.

18. The previous scenario can be modeled by a function: $V = 24(1.5)^h$, if V is the number of views after h hours. Suppose a business earns a profit of $700 this month and their profit is projected to double every month. Write a function that models the profit, P, after m months of doubling.

19. A savings account has an initial value of $1,200 and grows by 4% each year. Write a function that models the value of the account, V, after it has been growing for t years.

20. A painting is worth $4,000 in 2020 and its value increases by 12% each year.

    a. Write a function, $p(t)$, that models the painting's value t years after 2020.

    b. Graph the function on a graphing calculator and use the graph to determine when the painting will be worth $7,000. To do this, graph the 2 functions below and find their intersection point. Round to the nearest year.

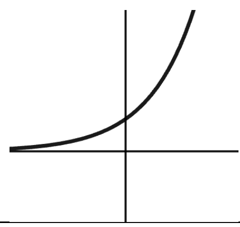
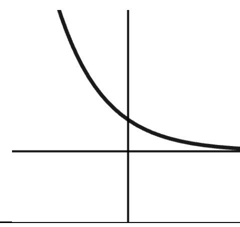
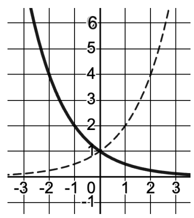
    $y_1 = 4000(1.12)^x$

    $y_2 = 7000$

21. A car is bought for $27,000 in 2010 and decreases in value by 8% per year. It is eventually sold to someone else for $21,000. In what year was it sold for $21,000? Use a graphing calculator.

SAMPLE PAGES

Answer Key

1.	increase: multiplying by 1.5 makes the y-values increase as x increases
2.	decrease: multiplying by 0.5 makes the y-values decrease as x increases
3.	$0 < b < 1$
4.	a. decay ($0.98 < 1$) b. growth ($1.02 > 1$) c. growth ($2.3 > 1$)
5.	a. growth ($0.6 + 0.6 \rightarrow 1.2$) b. decay ($1 - 0.3 \rightarrow 0.7$) c. decay ($2 - 1.5 \rightarrow 0.5$)
6.	a. growth  b. decay 
7.	a. $b = 0.5$; multiplying by 0.5 makes the y-values decrease b. 
8.	a. $2x^4 = 50x^3 \rightarrow x^4 = 25x^3 \rightarrow x = 25$ b. $54x = 2x^{-2} \rightarrow 27x = x^{-2} \rightarrow 27x = \frac{1}{x^2}$ $\rightarrow 27x^3 = 1 \rightarrow x^3 = \frac{1}{27} \rightarrow x = \frac{1}{3}$
9.	a. $k = 3$ (it is the horizontal asymptote) b. $1 = a(b)^3 \rightarrow a = \frac{1}{b^3}$ $7 = a(b)^5 \rightarrow a = \frac{7}{b^5}$ c. $b^5 = 4b^3 \rightarrow b^2 = 4 \rightarrow b = \pm 2$ ($b = 2$ because "b" can't be negative. Multiplying by a negative number repeatedly does not produce a continuous curve) d. $a = \frac{1}{8}$ e. $y = \frac{1}{8}(2)^x + 3$
10.	a. x^7 b. $\frac{12}{6(8)} \rightarrow \frac{12}{48} \rightarrow \frac{1}{4}$ c. $\frac{16}{5} \cdot \frac{1}{8} \rightarrow \frac{2}{5}$ d. $\frac{100}{x^3} \cdot \frac{1}{x^2} \rightarrow \frac{100}{x^5}$
11.	$k = 2 \rightarrow y = a(b)^x + 2$ plug in (1, 11) and (3, 3) $11 = a(b)^1 + 2 \rightarrow a = \frac{9}{b}$ $3 = a(b)^3 + 2 \rightarrow a = \frac{1}{b^3}$ $\frac{9}{b} = \frac{1}{b^3} \rightarrow 9b^3 = 1b \rightarrow b^2 = \frac{1}{9} \rightarrow b = \pm\frac{1}{3}$ $b = \frac{1}{3}$ because "b" can't be negative. Multiplying by a negative number repeatedly does not produce a continuous curve. $a = \frac{9}{\frac{1}{3}} \rightarrow a = 27$ $y = 27\left(\frac{1}{3}\right)^x + 2$
12.	$k = 4 \rightarrow y = a(b)^x + 4$ plug in (4, 16) and (1, 5.5) $16 = a(b)^4 + 4 \rightarrow a = \frac{12}{b^4}$ $5.5 = a(b)^1 + 4 \rightarrow a = \frac{1.5}{b}$ $\frac{1.5}{b} = \frac{12}{b^4} \rightarrow 1.5b^4 = 12b \rightarrow b^3 = 8 \rightarrow b = 2$ $a = \frac{1.5}{2} \rightarrow a = \frac{3}{4}$ $y = \frac{3}{4}(2)^x + 4$
13.	$224 = \frac{a}{b^3} \rightarrow 224b^3 = a$ $7 = ab^2 \rightarrow \frac{7}{b^2} = a$ $224b^3 = \frac{7}{b^2} \rightarrow 224b^5 = 7 \rightarrow b^5 = \frac{7}{224}$ $\rightarrow b^5 = \frac{1}{32} \rightarrow b = \frac{1}{2}$ $a = \frac{7}{b^2} \rightarrow a = \frac{7}{\left(\frac{1}{2}\right)^2} \rightarrow a = \frac{7}{\frac{1}{4}} \rightarrow a = 28$
14.	$y = a(b)^x - 3$ plug in $(-5, 15)$ and $(-3, -1)$ $15 = a(b)^{-5} - 3 \rightarrow a = 18b^5$ $-1 = a(b)^{-3} - 3 \rightarrow a = 2b^3$ $18b^5 = 2b^3 \rightarrow b^2 = \frac{1}{9} \rightarrow b = \pm\frac{1}{3}$ b must be positive $\rightarrow b = \frac{1}{3}$ $a = 2b^3 \rightarrow a = 2\left(\frac{1}{3}\right)^3 \rightarrow a = \frac{2}{27}$ $y = \frac{2}{27}\left(\frac{1}{3}\right)^x - 3$
15.	20%
16.	10%
17.	a. $24(1.5)^3$ b. $24(1.5)^h$
18.	$P = 700(2)^m$

Section 5

Compound Interest

Use this page for taking notes or anything else that helps you learn.

Introduction to Compound Interest

When a scenario involves "simple interest," the interest is added once a year. For example, if a $1,000 investment has an interest rate of 4%, its value will increase 4% per year. How much does it earn if it is in the account for only 6 months? 2%? If the interest is only added at the year's end, the investment earns nothing if it is taken out of the account earlier. If that seems unfair, you will be relieved to know that interest is usually added more often than once a year.

1. For example, suppose a yearly interest rate is 4%, but it is added to the account 4 times a year. In this case, the 4% gets split into 4 equal parts of 1% each. Four times a year, the account value increases by 1%. This method of splitting a yearly interest rate into smaller parts is called compounding.

    a. If the yearly interest rate is 8% and the interest is added to the account 4 times per year, by what percent does the account increase each time interest is added?

    b. If the interest rate is 3% per year and the interest is added 12 times per year, by what percent does the account increase each time?

2. How many times per year is interest added to the account if it is compounded as shown?

    a. daily b. quarterly c. weekly d. hourly

3. $20,000 is invested at an annual interest rate of 12%, compounded monthly.

    a. What is the investment's value after 1 month?

    b. What is the investment's value after 3 months?

4. $24,000 is invested at an annual interest rate of 8%, compounded weekly. What is the balance after 20 weeks?

5. An investment of $13,500 has an annual interest rate of 6.45%, compounded quarterly. What is the balance after 2.5 years?

6. When interest is compounded n times per year, the value of an initial investment after t years can be modeled by the formula shown.

$$V = P\left(1+\frac{r}{n}\right)^{nt}$$

V = account's ______________ after t years

P = __________________ (initial value)

r = annual interest ___________ (as a decimal)

t = _________ in years

n = number of times the interest is ____________________ each year

7. $10,000 is invested at an annual interest rate of 4.2%, compounded monthly. What is the balance after 5 years?

8. An initial investment of $1,000 has a yearly interest rate of 6% per year, compounded quarterly.

a. What is the value of the account after 1 year?

b. By what percent does the account increase after 1 year?

9. If a yearly interest rate is 6%, but it is compounded monthly, the account increases by a little more than 6% each year. Why does this happen?

10. $4,000 is invested in an account with interest compounded weekly. It grows to $6,000 in 3 years. What was the interest rate? Round to the nearest tenth of a percent.

Continuously Compounded Interest

11. When interest is compounded n times per year, the effect is different for different n-values. Make a guess. Is it better to compound the interest 4 times or 40 times per year?

SAMPLE PAGES

Answer Key

1.	a. $0.08 \div 4 = 0.02 \rightarrow 2\%$ b. $0.03 \div 12 = 0.0025 \rightarrow 0.25\%$
2.	a. 365 b. 4 c. 52 d. $24 \cdot 365 \rightarrow 8{,}760$
3.	a. $0.12 \div 12 = 0.01 \rightarrow$ it grows 1% per month $20000(1.01) \rightarrow \$20{,}200$ b. $20000(1.01)^3 \rightarrow \$20{,}606.02$
4.	$24000\left(1+\frac{0.08}{52}\right)^{20} \rightarrow \$24{,}749.35$
5.	2.5 years is $2.5 \cdot 4 \rightarrow 10$ quarters $13500\left(1+\frac{0.0645}{4}\right)^{10} \rightarrow \$15{,}841.82$
6.	$V =$ value $P =$ principle $r =$ rate $t =$ time $n =$ compounded
7.	$10000\left(1+\frac{0.042}{12}\right)^{(12)(5)} \rightarrow \$12{,}332.26$
8.	a. $1000\left(1+\frac{0.06}{4}\right)^{(4)(1)} \rightarrow \$1{,}061.36$ b. $\frac{1061.36-1000}{1000} \rightarrow 0.06136 \rightarrow 6.14\%$
9.	Each month, the account increases in value by $\frac{1}{12}$ of 6%. When the interest is added, the monthly growth increases as well because the interest added each month also grows by $\frac{1}{12}$ of 6%.
10.	$4000\left(1+\frac{r}{52}\right)^{(52)(3)} = 6000$ $\rightarrow \left(1+\frac{r}{52}\right)^{156} = 1.5 \rightarrow 1+\frac{r}{52} = \sqrt[156]{1.5}$ $\rightarrow \frac{r}{52} = \sqrt[156]{1.5} - 1 \rightarrow r = 52(0.0026)$ $\rightarrow r = 0.1353 \rightarrow$ the rate is 13.5%
11.	40 times
12.	a. \$10613.64 b. \$10617.89 c. \$10618.32

13.	$n = 24 \cdot 365$ $1\left(1+\frac{1}{8760}\right)^{8760(1)} \rightarrow \$2.72$
14.	$e = 2.72$ $e = 2.72$
15.	$V =$ value $P =$ principle $r =$ rate $t =$ time
16.	a. $0.3 \rightarrow 30\%$ b. $0.06 \rightarrow 6\%$ c. 4.03%
17.	$V = 8{,}000e^{(0.07)(5)} \rightarrow \$11{,}352.54$
18.	$V = 200{,}000e^{(0.08)\left(\frac{1}{365}\right)} \rightarrow \$200{,}043.84$ $V = 200{,}000e^{(0.08)(20)} \rightarrow \$990{,}606.48$
19.	$V = 125{,}000e^{(0.057)(3.8)} \rightarrow \$155{,}230.91$
20.	Solve: $2500e^{0.028t} = 5000$ Find the intersection of $y = 2500e^{0.028t}$ and $y = 5000 \rightarrow (24.755, 5000)$ It will double in 24.8 years.
21.	Solve: $2500e^{0.028t} = 3000e^{0.0236t}$ Find the intersection of $y = 2500e^{0.028t}$ and $y = 3000e^{0.0236t} \rightarrow (41.437, 7976.65)$ Kyle's account will be larger than Linda's after 41.4 years.
22.	Solve two separate equations. Equation 1: $6500e^{0.039t} = 10000$ Find the intersection of $y = 6500e^{0.039t}$ and $y = 10000$ $\rightarrow (11.046, 10000) \rightarrow t = 11.046$ years Equation 2: $7150e^{0.0308t} = 10000$ Find the intersection of $y = 7150e^{0.0308t}$ and $y = 10000$ $\rightarrow (10.892, 10000) \rightarrow t = 10.892$ years $11.046 - 10.892 = 0.154$ years $0.154(365) = 56$ days Joni reaches \$10,000 first by 56 days.
23.	a. $V = P(1+r)^t$ b. $V = P\left(1+\frac{r}{n}\right)^{nt}$ c. $V = Pe^{rt}$

CONTENTS

Unit 8

Logarithmic Functions, Equations & Properties

Section 1

The Logarithm

Use this page for taking notes or anything else that helps you learn.

Introduction to Logarithms

1. In a previous lesson, you learned about exponential equations.

   a. If $3^x = 9$, what is x?

   b. What is the exponent if $3^x = 10$? Make a quick guess.

> If $3^x = 10$, then x is 2.0959.... (the digits keep going). To find this value with a calculator, you need to learn a new word, **logarithm**, which means the exponent that makes a base become a specific value.

2. In the expression 3^2, the 3 is a base and the 2 is an exponent. Identify the base in each expression.

a. 4^0	b. $(-2)^4$	c. $\left(\frac{1}{3}\right)^{0.2x}$	d. x^5	e. $(x+2)^7$
base:	base:	base:	base:	base:

> Consider the statement $3^2 = 9$. The exponent that makes a base of 3 become 9 is 2. Read each statement below to see how the word "logarithm" is used to represent an exponent.
>
> Using "exponent": The exponent that makes a base of 3 become 9 is 2.
>
> Using "logarithm": The logarithm that makes a base of 3 become 9 is 2.
>
> Using logarithm notation: log base 3 of 9 is 2 → $\log_3 9 = 2$

3. A logarithm is an exponent. To help you become familiar with the word logarithm, read each statement below and its shortened form. Then, fill in the blanks.

   a. The exponent (logarithm) that makes a base of 5 become 25 is 2 → $\log_5 25 =$ ___

   b. The logarithm that makes a base of 3 become 27 is 3 → log__ ___ = ___

   c. The logarithm that makes a base of 4 become 1 is 0 → log__ ___ = ___

4. The expression $\log_6 36 = 2$ is pronounced "log base 6 of 36 is 2." Practice this by completing each statement below.

   a. $\log_7 49 = 2$ → log base 7 of 49 is _____

   b. $\log_2 16 = 4$ → log base _____ of 16 is _____

   c. $\log_5 1 = 0$ →

5. Rewrite each logarithm statement in exponential form. The first one is done for you.

a. $\log_5 25 = 2$	b. $\log_3 27 = 3$	c. $\log_4 1 = 0$	d. $\log_5 5 = 1$
$5^2 = 25$			

Switching Between Exponential & Logarithmic Form

To convert between exponential form and logarithmic form, use the circular relationship shown.

logarithmic form to exponential: $\log_5 25 = 2 \rightarrow 5^2 = 25$

exponential form to logarithmic: $4^3 = 64 \rightarrow \log_4 64 = 3$

6. Each statement below is in logarithmic form. Rewrite it in exponential form.

a. $\log_6 36 = 2$ b. $\log_{10} 1{,}000 = 3$ c. $\log_2 32 = 5$

7. Switch the form of each statement shown.

a. $\log_{11} 1 = 0$ b. $\log_{81} 9 = \frac{1}{2}$ c. $\log_x \left(\frac{1}{x^2}\right) = -2$

8. Each statement is in exponential form. Rewrite it in logarithmic form.

a. $3^2 = 9$ b. $2^4 = 16$ c. $10^3 = 1{,}000$

9. Switch the form of each statement shown.

a. $4^{-1} = \frac{1}{4}$ b. $(0.5)^2 = 0.25$ c. $A^B = C$

10. Without a calculator, what is $\log_{10}(10)$? Why?

11. Why is $\log_x(1)$ equal to 0?

12. Without a calculator, what is the value of each expression below?

a. $\log_7 49$ b. $\log_6 36$ c. $\log_2 8$

Answer Key

1.	a. $3^2 = 9$ b. $3^{2.1} \approx 10$
2.	a. 4 b. -2 c. $\frac{1}{3}$ d. x e. $x + 2$
3.	a. $\log_5 25 = 2$ b. $\log_3 27 = 3$ c. $\log_4 1 = 0$
4.	a. log base 7 of 49 is 2 b. log base 2 of 16 is 4 c. log base 5 of 1 is 0
5.	b. $3^3 = 27$ c. $4^1 = 1$ d. $5^1 = 5$
6.	a. $6^2 = 36$ b. $10^3 = 1000$ c. $2^5 = 32$
7.	a. $11^0 = 1$ b. $81^{1/2} = 9$ c. $x^{-2} = \frac{1}{x^2}$
8.	a. $\log_3 9 = 2$ b. $\log_2 16 = 4$ c. $\log_{10} 1000 = 3$
9.	a. $\log_4 \frac{1}{4} = -1$ b. $\log_{0.5} 0.25 = 2$ c. $\log_A C = B$
10.	$\log_{10}(10) = 1$ because $10^1 = 10$
11.	$\log_x(1) = 0$ because $x^0 = 1$
12.	a. $\log_7 49 = 2$, because $7^2 = 49$ b. $\log_6 36 = 2$, because $6^2 = 36$ c. $\log_2 8 = 3$, because $2^3 = 8$
13.	a. -1 b. 0 c. 6
14.	a. log10 b. log37
15.	a. x b. x c. -1
16.	a. $\log 1 = 0$ b. $\log(10{,}000) = 4$
17.	a. $\log 100 = 2$ b. $\log\left(\frac{1}{10}\right) = -1$ c. $\log 63 \approx 1.8$

18.	$e \approx 2.72$
19.	a. ln10 b. ln2 c. ln$e$
20.	a. $\ln e = 1$ b. $\ln\left(e^2\right) = 2$
21.	a. $\ln(7.39) \approx 2$ b. $\ln\left(\frac{1}{e^3}\right) = -3$ c. $\ln(3009) \approx 5.7$
22.	a. $10^x = 13$ b. $e^3 = x$ c. $10^4 = x$
23.	a. $\log(x) = 3.2$ b. $\ln 7 = 4x$ c. $\log(x) = \frac{1}{3}$
24.	a. $2^3 = x \rightarrow x = 8$ b. $5^2 = x \rightarrow x = 25$ c. $9^{\frac{1}{2}} = x \rightarrow x = 3$
25.	a. $x^3 = 8 \rightarrow x = 2$ b. $x^{\frac{1}{2}} = 9 \rightarrow x = 81$ c. $x^{\frac{1}{3}} = 2 \rightarrow x = 8$
26.	a. $\log_2(8) = x - 5 \rightarrow 3 = x - 5 \rightarrow x = 8$ b. $\log(100) = 4x \rightarrow 2 = 4x \rightarrow x = \frac{1}{2}$
27.	a. $\log_7 x = 1 \rightarrow 7^1 = x \rightarrow x = 7$ b. $\log_4(x + 3) = 3 \rightarrow 4^3 = x + 3 \rightarrow x = 61$ c. $3^x = 27 \rightarrow \log_3(27) = x \rightarrow x = 3$
28.	a. $\log_5 y = x - 2 \rightarrow \log_5 y + 2 = x$ b. $y - 2 = 7^x \rightarrow \log_7(y - 2) = x$ c. $2^y = x + 6 \rightarrow 2^y - 6 = x$ d. $y + 4 = \log_3(x) \rightarrow 3^{y+4} = x$
29.	a. $f^{-1}(x) = \log_4(x) + 2$ b. $f^{-1}(x) = 2^x + 10$
30.	a. $f^{-1}(x) = \log(x + 6)$ b. $f^{-1}(x) = e^{x+3}$

Section 2

Properties of Logarithms

Use this page for taking notes or anything else that helps you learn.

Introduction to Properties

1. In this section, you will learn about properties of logarithms. Before you discover these properties, practice evaluating logarithmic expressions. Simplify each expression without a calculator.

a. $\log_2 2$ b. $\log_7 49$ c. $\log_8 1$

2. Without a calculator, determine the value of each expression below.

a. $\log_2 \left(\frac{1}{32}\right)$ b. $\log_{27}(3)$ c. $\log_8 4$

3. Before you learn about logarithm properties, try to recall 3 exponent properties: the Product Rule, Quotient Rule and Power Rule.

a. Product Rule b. Quotient Rule c. Power Rule

$x^9 \cdot x^2 =$ $\frac{x^7}{x^3} =$ $\left(x^2\right)^3 =$

The Product Rule for Logarithms

4. The Product Rule for exponents is this: when you multiply like bases, add the exponents. For example, $x^3 \cdot x^4 = x^{3+4} = x^7$. Use the Product Rule for exponents to simplify each expression.

a. $y^6 \cdot y^3 =$ b. $3^{10} \cdot 3 =$ c. $5x^{-2} \cdot 4x^8 =$

The Product Rule for logarithms also involves multiplying and adding.
Two examples of the Product Rule for logarithms are shown.

$\log_2(5) + \log_2(3) = \log_2(15)$ $\log_3(8) + \log_3(5) = \log_3(40)$

5. Even though you haven't learned why this property exists, use the examples in the previous scenario to guess the value of each expression below.

a. $\log_9(5) + \log_9(2) =$______________ b. $\log_5(4) + \log_5(6) = \log_5($____$)$

6. To see why the Product Rule exists, fill in the blanks below and try to look for patterns.

a. $\log_2 4 + \log_2 8$ b. $\log_2($_____$) = 5$

____ + ____

7. The expressions "$\log_2 4 + \log_2 8$" and "$\log_2 32$" both equal 5. Using the transitive property, it follows that $\log_2 4 + \log_2 8 = \log_2 32$. Consider another example. Fill in the blanks.

a. $\log_3 9 + \log_3 3$ b. $\log_3($_____$) = 3$ c. $\log_3 9 + \log_3 3 = \log_3($_____$)$

____ + ____

Answer Key

1.	a. 1 b. 2 c. 0
2.	a. -5 b. $\frac{1}{3}$ c. $\frac{2}{3}$
3.	a. x^{11} b. x^4 c. x^6
4.	a. y^9 b. 3^{11} c. $20x^6$
5.	a. $\log_9(5\cdot 2) \rightarrow \log_9(10)$ b. $\log_5(4\cdot 6) \rightarrow \log_5(24)$
6.	a. $2+3 \rightarrow 5$ b. 32
7.	a. $2+1 \rightarrow 3$ b. 27 c. 27
8.	a. $\log_3 20$ b. $\log_5 14$
9.	$\log_b XY$
10.	a. $\log_2 30$ b. $\log_3 2A$ c. $\log 100 \rightarrow 2$
11.	$\log_b X + \log_b Y$
12.	a. $\log_2 6 + \log_2 x$ b. $\log_3 9 + \log_3 f \rightarrow 2 + \log_3 f$ c. $\log_4 4 + \log_4 y \rightarrow 1 + \log_4 y$
13.	a. $\log_5 25 + \log_5 a + \log_5 b$ $\rightarrow 2 + \log_5 a + \log_5 b$ b. $\ln 7 + \ln e \rightarrow \ln 7 + 1$ c. $\log 100 + \log 10^x \rightarrow 2 + x$
14.	a. $\log_3(6x)$ b. $\log_4(x^2+5x)$
15.	$\log_2(5x) = 3 \rightarrow 2^3 = 5x \rightarrow x = \frac{8}{5}$
16.	a. $\log_4(8m) = 2 \rightarrow 4^2 = 8m \rightarrow m = 2$ b. $\log_3(4y-8) = 1 \rightarrow 3^1 = 4y - 8$ $\rightarrow y = \frac{11}{4}$ or 2.75
17.	$\log_4(x^2-3x) = 1 \rightarrow x^2 - 3x = 4$ $\rightarrow x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0$ $\rightarrow x = 4$ or -1 Check the solutions with the original equation. Although factoring produces 2 solutions, $x = 4$ is the only valid solution, because $\log_4(-1)$ is undefined.
18.	$\log_6(x^2+5x) = 1 \rightarrow x^2 + 5x = 6^1$ $\rightarrow x^2 + 5x - 6 = 0 \rightarrow (x-1)(x+6) = 0$ $\rightarrow x = 1, -6$ → check solutions $x = 1$ is the only solution, because $\log_6(-6)$ is undefined.
19.	You can subtract the exponents when you divide like bases.
20.	a. x^5 b. x^{18} c. $3x^{10}$
21.	$\log_b X - \log_b Y = \log_b\left(\frac{X}{Y}\right)$

22.	a. $\log_3\left(\frac{20}{4}\right) \rightarrow \log_3 5$ b. $\log_5\left(\frac{12}{2}\right) \rightarrow \log_5 6$
23.	a. 3 b. 1 c. $3-1 \rightarrow 2$ d. 9
24.	a. $\log_7\left(\frac{14}{2}\right) \rightarrow \log_7 7 \rightarrow 1$ b. $\log\left(\frac{200}{2}\right) \rightarrow \log 100 \rightarrow 2$ c. $\log_8\left(\frac{20}{10}\right) \rightarrow \log_8 2 \rightarrow \frac{1}{3}$
25.	a. $\log_3\left(\frac{14}{x}\right)$ b. $\log_4\left(\frac{x}{x-6}\right)$
26.	$\log_b\left(\frac{X}{Y}\right) = \log_b X - \log_b Y$
27.	a. $\log_5 x - \log_5 5 \rightarrow \log_5 x - 1$ b. $\log_6 36 - \log_6 y \rightarrow 2 - \log_6 y$ c. $\log_7 1 - \log_7 w \rightarrow 0 - \log_7 w \rightarrow -\log_7 w$
28.	a. $\log x - \log 10^2 \rightarrow \log x - 2$ b. $\ln e^3 - \ln y \rightarrow 3 - \ln y$
29.	$\log_3\left(\frac{x}{2}\right) = 2 \rightarrow 3^2 = \frac{x}{2} \rightarrow x = 18$
30.	a. $\log_3\left(\frac{36}{m}\right) = 3 \rightarrow 3^3 = \frac{36}{m} \rightarrow m = \frac{4}{3}$ b. $\log_2\left(\frac{3-y}{10}\right) = -1 \rightarrow 2^{-1} = \frac{3-y}{10}$ $\rightarrow 5 = 3 - y \rightarrow y = -2$
31.	a. $\log_2\left(\frac{3-x}{x}\right) = -3 \rightarrow 2^{-3} = \frac{3-x}{x}$ $\rightarrow \frac{1}{8} = \frac{3-x}{x} \rightarrow 8(3-x) = x$ $\rightarrow 24 - 8x = x \rightarrow x = \frac{8}{3}$ b. $\log_x\left(27\cdot\frac{1}{9}\right) = -1 \rightarrow \log_x(3) = -1$ $\rightarrow x^{-1} = 3 \rightarrow \frac{1}{x} = 3 \rightarrow x = \frac{1}{3}$
32.	a. x^6 b. $27^{2x/3}$ c. x^{2x+2} d. 5^{3x^2-6x}
33.	a. $3x$ b. $2x^3$ c. $2\cdot 5^x$ d. $3\cdot 7^y$
34.	a. $2\log(x)$ b. $3\log(y)$ c. $4\log(z)$ d. $5\log(w)$
35.	$P\log_b(M)$
36.	a. $2\log_5(x)$ b. $3\log_6(x)$ c. $x\log_2(2)$ d. $B\log(10)$
37.	a. $\log_4(5^2)$ b. $\log_2(y^3)$ c. $\log_4\left(49^{1/2}\right)$
38.	a. $\log(x^7)$ b. $\log_7(y^4) \rightarrow 4\log_7(y)$ c. $\log_6\left(7^{1/3}\right) \rightarrow \frac{1}{3}\log_6(7)$
39.	a. $2^{3\log(10)} \rightarrow 2^{3\cdot 1} \rightarrow 8$ b. $49^{\frac{1}{2}\ln(e)} \rightarrow 49^{\frac{1}{2}\cdot 1} \rightarrow 7$

Section 3

Solving Logarithmic Equations

Use this page for taking notes or anything else that helps you learn.

Simple Logarithmic Equations Part 1: $\log_b(X) = Y$

1. When an equation has the form $\log_b(X) = Y$, you can often solve it by switching from logarithmic to exponential form. Switch each equation to exponential form. Do not solve the equation.

   a. $\log_5(4x) = 2$ b. $\log_x(x+2) = 2$ c. $\ln(2x) = 5$ d. $\log(5x+1) = -2$

2. Each equation shown needs one more operation done to isolate $\log_b(X)$, to make it $\log_b(X) = Y$. Perform this first step and then switch forms to isolate x in each equation. Simplify the result.

   a. $\log_7(x) + 12 = 10$ b. $2\log_{25} x = 1$ c. $\dfrac{-3}{\log_4 x} = 1$

Check solutions when solving logarithmic equations.
The expression $\log_b(x)$ is undefined if $x \leq 0$. For example, $\log_2(0)$ and $\log_3(-9)$ are undefined.
Logarithms with negative bases are also undefined. For example, $\log_{-2}(4)$ and $\log_{-3}(9)$ are undefined.

Advanced Logarithmic Equations Part 1: $\log_b(X) = Y$

3. Each equation shown is one step away from this form: $\log_b(X) = Y$. Apply what you have learned about properties to solve the equation.

   a. $\log_4(x) + \log_4(5) = 2$ b. $\log_6(2) - \log_6(x) = -2$

4. Solve each equation.

   a. $\log_2(3) + \log_2(x-2) = 3$ b. $\log_5(2x+3) - \log_5(x) = 0$

Advanced Logarithmic Equations Part 2: $\log_b(X) = \log_b(Y)$

10. Each equation shown is one step away from this form: $\log_b(X) = \log_b(Y)$. Apply what you have learned about properties to solve the equation.

a. $\log_8(6) + \log_8(3x) = \log_8(9)$

b. $\ln(x) - \ln(3) = \ln(5)$

11. Solve each equation.

a. $\log_3(x) + \log_3(5) = \log_3(2x + 21)$

b. $\log(3) - \log(x - 7) = \log(6)$

12. Solve each equation.

a. $\log_5(24) - \log_5(x - 2) = \log_5(x + 3)$

b. $\ln\left(3x^2 - 8\right) = \ln(5x)$

Logarithmic Equations Part 3: $\log_a(X) = \log_b(Y)$

13. Consider this equation: $\log_3(x - 3) = \log_9(16)$. Since the bases are different, how can you solve this equation? Think about it briefly and then keep reading.

You can solve the equation if you can make the bases equal. To learn how to change a base, consider each expression below. Use a calculator to find the value of each expression.

$\log_2(3)$ $\log_4(9)$ $\log_{2^3}(3^3)$ $\log_{\sqrt{2}}(\sqrt{3})$

Answer Key

1.	a. $5^2 = 4x$ b. $x^2 = x + 2$ c. $e^5 = 2x$ d. $10^{-2} = 5x + 1$
2.	a. subtract 12 → $\log_7(x) = -2$ → $7^{-2} = x$ → $x = \frac{1}{49}$ b. divide by 2 → $\log_{25} x = \frac{1}{2}$ → $25^{\frac{1}{2}} = x$ → $\sqrt{25} = x$ → $5 = x$ c. multiply by $\log_4 x$ → $-3 = \log_4 x$ → $4^{-3} = x$ → $x = \frac{1}{64}$
3.	a. Product Rule → $\log_4(5x) = 2$ → $4^2 = 5x$ → $x = \frac{16}{5}$ b. Quotient Rule → $\log_6\left(\frac{2}{x}\right) = -2$ → $6^{-2} = \frac{2}{x}$ → $x = 72$
4.	a. $\log_2\big(3(x-2)\big) = 3$ → $\log_2(3x-6) = 3$ → $2^3 = 3x - 6$ → $x = \frac{14}{3}$ b. $\log_5\left(\frac{2x+3}{x}\right) = 0$ → $5^0 = \frac{2x+3}{x}$ → $1 = \frac{2x+3}{x}$ → $x = 2x + 3$ → $x = -3$ Check the solution: $\log_5(-3)$ is undefined, so the equation has no solution.
5.	a. $\log_{27}\left(x^2 + 2x\right) = \frac{1}{3}$ → $27^{\frac{1}{3}} = x^2 + 2x$ → $3 = x^2 + 2x$ → $0 = x^2 + 2x - 3$ → $(x+3)(x-1)$ → $x = -3$ or 1 Since $\log_{27}(-3)$ is undefined, the only solution is $x = 1$ b. $\log_{16}\left(x^2\right) - \log_{16}(25) = -1$ → $\log_{16}\left(\frac{x^2}{25}\right) = -1$ → $16^{-1} = \frac{x^2}{25}$ → $\frac{1}{16} = \frac{x^2}{25}$ → $16x^2 = 25$ → $x^2 = \frac{25}{16}$ → $x = \pm\frac{5}{4}$ → $\log_{16}\left(-\frac{5}{4}\right)$ is undefined, so the only solution is $x = \frac{5}{4}$
6.	a. switch forms: $8^{\frac{2}{3}} = \log_3(x)$ → $4 = \log_3(x)$ → $3^4 = x$ → $x = 81$ b. switch forms: $16^{\frac{1}{4}} = \log_x(9)$ → $2 = \log_x(9)$ → $x^2 = 9$ → $x = \pm 3$ → A logarithm's base must be positive so the only solution is $x = 3$.
7.	If $\log_b(X) = \log_b(Y)$, then $X = Y$.

8.	a. $x = 7$ b. $x + 5 = 2x - 4$ → $x = 9$ c. $x^2 = 36$ → $x = \pm 6$ All solutions are valid when checked.
9.	a. $x^2 - 9x = 10$ → $x^2 - 9x - 10 = 0$ → $x = 10$ or -1 Both solutions are valid. b. $x^2 + 8 = 108$ → $x^2 = 100$ → $x = \pm 10$ Both solutions are valid.
10.	a. $\log_8(18x) = \log_8(9)$ → $18x = 9$ → $x = \frac{1}{2}$ b. $\ln\left(\frac{x}{3}\right) = \ln(5)$ → $\frac{x}{3} = 5$ → $x = 15$
11.	a. $\log_3(5x) = \log_3(2x + 21)$ → $5x = 2x + 21$ → $x = 7$ b. $\log\left(\frac{3}{x-7}\right) = \log(6)$ → $\frac{3}{x-7} = 6$ → $3 = 6x - 42$ → $45 = 6x$ → $x = 7.5$
12.	a. $\log_5\left(\frac{24}{x-2}\right) = \log_5(x+3)$ → $\frac{24}{x-2} = x + 3$ → $24 = (x+3)(x-2)$ → $24 = x^2 + x - 6$ → $0 = x^2 + x - 30$ → $0 = (x+6)(x-5)$ → $x = -6$ or 5 → $\log_5(-6-2)$ → $\log_5(-8)$ is undefined, so the only solution is $x = 5$ b. $3x^2 - 8 = 5x$ → $3x^2 - 5x - 8 = 0$ → $(3x-8)(x+1) = 0$ → $x = \frac{8}{3}$ or -1 → $\ln\big(5(-1)\big)$ → $\ln(-5)$ is undefined, so the only solution is $x = \frac{8}{3}$
13.	All 4 expressions are equal to 1.58496....
14.	a. $\log_9(25)$ b. $\log_{16}(49)$ c. $\log_5(3)$
15.	a. $\log_{81}(36)$ b. $\log_7\left(\frac{1}{8}\right)$ c. $\log_2(5)$
16.	a. $\log_4 x = \log_4 7$ → $x = 7$ b. $\log_9(x+5) = \log_9(4)$ → $x = -1$ c. $\log_8\left(x^3\right) = \log_8(25x)$ → $x^3 = 25x$ → $x^3 - 25x = 0$ → $x\left(x^2 - 25\right) = 0$ → $x(x+5)(x-5) = 0$ → $x = 0, 5$ or -5 → $\log_8(0)$ and $\log_8(-125)$ are both undefined, so the only solution is $x = 5$.
17.	a. $\log_2(10)$ b. $\log(3)$ c. $\ln(75)$
18.	a. 1.996 b. 3.011 c. 54.598 d. 1.414
19.	a. $7^x = 59$ → $x = \log_7(59)$ b. $e^x = 3$ → $x = \ln(3)$ c. $14 = 10^x$ → $x = \log(147)$
20.	Solve: $300 = 200(1.03)^m$ → $1.5 = (1.03)^m$ → $m = \log_{1.03}(1.5)$ → $m \approx 13.717$ The population is 300 after 13.7 months.

Section 4

Graphing Logarithmic Functions

Use this page for taking notes or anything else that helps you learn.

The Graph of a Logarithmic Function

1. You can use what you know about exponents to draw a logarithmic function's graph. First, recall the exponential function $y = 2^x$. Find the output for each input shown.

   a. $x = -1$ b. $x = 0$ c. $x = 1$ d. $x = 2$ e. $x = 4$

   $y = 2^{-1}$ $y = 2^0$

   $y =$ $y =$

2. Now consider the function $y = \log_2(x)$. Find the output for each input shown.

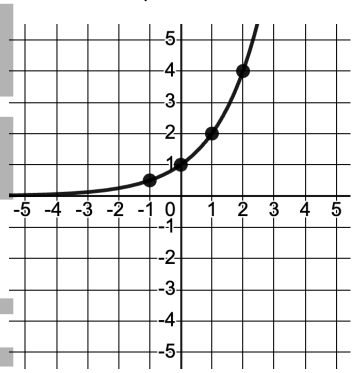
   a. $x = 0$ b. $x = \frac{1}{2}$ c. $x = 1$ d. $x = 2$ e. $x = 4$

   $y = \log_2(0)$

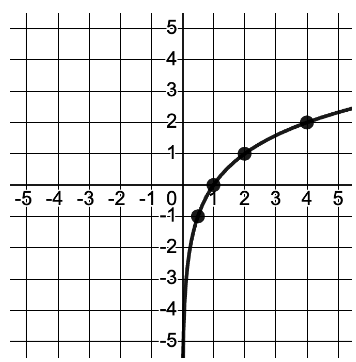
   $y =$

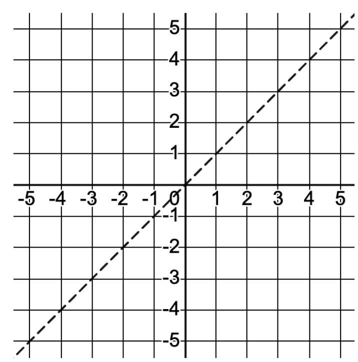
3. The functions $y = 2^x$ and $y = \log_2(x)$ are inverses. They show the same information, but in different ways. Both functions are shown below in separate graphs. In the third graph, draw both functions. Plot the same 4 points that are shown before drawing each curve.

$y = 2^x$



$y = \log_2(x)$





The graph of $y = 2^x$ has <u>only positive y-values</u> because no exponent makes a positive base either 0 or negative. The graph of $y = \log_2(x)$ has <u>only positive x-values</u> for the same reason.

4. Write the domain and range of each function. Then, identify each intercept, if it exists.

   a. $y = 2^x$

   domain: range:

   x-intercept: y-intercept:

   b. $y = \log_2(x)$

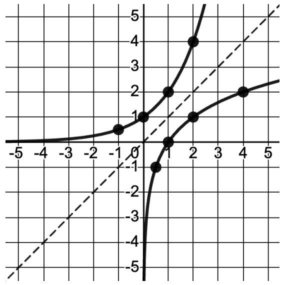
   domain: range:

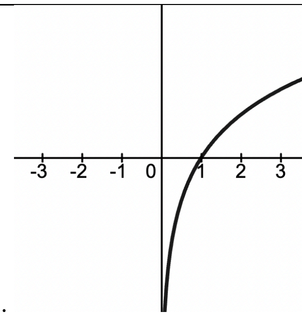
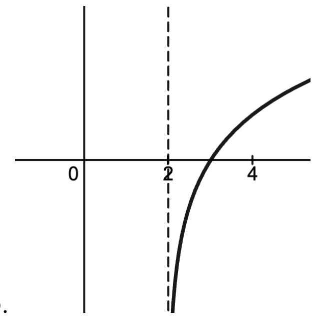
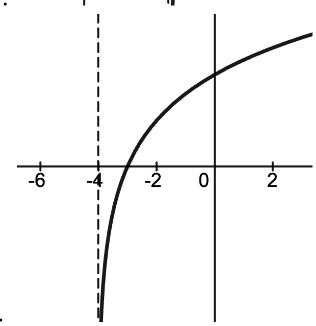
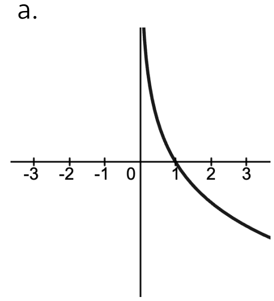
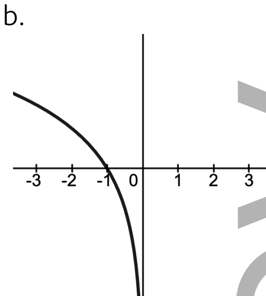
   x-intercept: y-intercept:

5. The graph of $y = 2^x$ has a horizontal asymptote along the x-axis because no x-value makes 2^x negative or 0. The graph of $y = \log_2(x)$ has a vertical asymptote along the ____________.

   What is the equation of the vertical asymptote of $y = \log_2(x)$?

Answer Key

1.	a. $y = \frac{1}{2}$ b. $y = 2^0 = 1$ c. $y = 2^1 = 2$ d. $y = 2^2 = 4$ e. $y = 2^4 = 16$
2.	a. $\log_2(0)$ is undefined b. $\log_2\left(\frac{1}{2}\right) = -1$ c. $y = \log_2(1) = 0$ d. $y = \log_2(2) = 1$ e. $y = \log_2(4) = 2$
3.	
4.	a. domain: $(-\infty, \infty)$ range: $(0, \infty)$ x-intercept: none y-intercept: $(0, 1)$ b. domain: $(0, \infty)$ range: $(-\infty, \infty)$ x-intercept: $(1, 0)$ y-intercept: none
5.	y-axis equation: $x = 0$
6.	a. Changing x to "$x - 3$" moves the function right 3. The vertical asymptote is $x = 3$. b. shift left 7: $x = -7$ c. shift right 4: $x = 4$
7.	a. $y = \log_2\big(3(x-1)\big) \rightarrow$ shift right 1 The vertical asymptote is $x = 3$. b. $y = \log_2\big(7(x+2)\big) \rightarrow$ shift left 2 vertical asymptote: $x = -2$ c. $y = \log_2\left(4\left(x - \frac{1}{2}\right)\right) + 6 \rightarrow$ shift right $\frac{1}{2}$ vertical asymptote: $x = \frac{1}{2}$
8.	a. $(x-5) \rightarrow$ shift right 5 The vertical asymptote is $x = 5$. b. $y = \log_2\big(-(x-4)\big) \rightarrow$ shift right 4 vertical asymptote: $x = -2$ c. $y = \log_2(-x+2) + 3$ $y = \log_2\big(-(x-2)\big) + 3 \rightarrow$ shift right 2 vertical asymptote: $x = 2$

9.	 a.  b.  c.
10.	a.  b.  c.