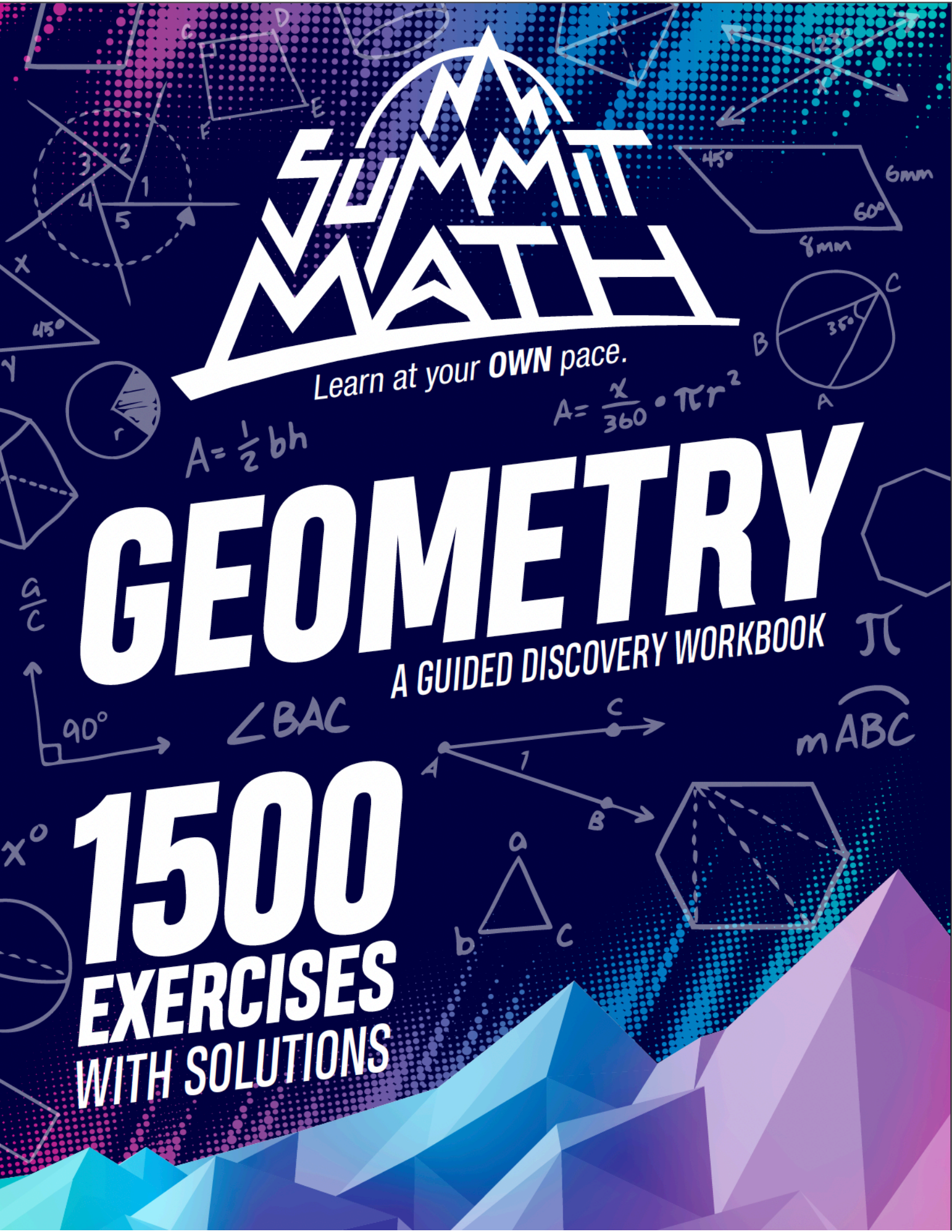
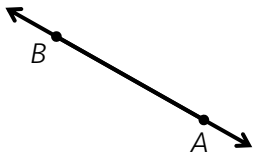

SUMMIT MATH
Learn at your OWN pace.
GEOMETRY
A GUIDED DISCOVERY WORKBOOK
1500 EXERCISES WITH SOLUTIONS

CONTENTS

Unit 1 Summary

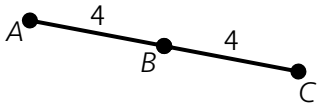
The key terms and concepts in this Unit:

- **Key Figures:**
  - A **point** is represented by a dot and named with a letter.
  - A **segment** ($\overline{AB}$) connects two endpoints.
  - A **ray** ($\overrightarrow{AB}$) starts at a point and extends infinitely in one direction.
  - A **line** ($\overleftrightarrow{AB}$) extends in both directions without ending.

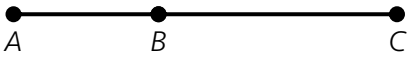


- **Collinear Points:** Three or more points that lie on the same line. Unless the line is vertical, the points can be represented by a linear equation in Slope-Intercept Form: $y = mx + b$.
  - Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

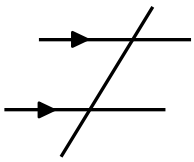
- **Midpoints & Distance:**
  - A **midpoint** is the exact middle point of a segment, **bisecting** it into two **congruent** parts. It is the average of the *x*- and *y*-values of the endpoints.
    - Midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



  - The **Distance Formula** is used to calculate the length between any two points.
    - $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
  - **Segment Addition Postulate:** $AB + BC = AC$



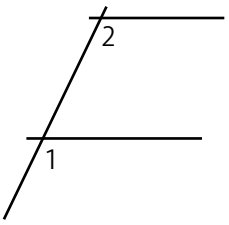
- **Angles:** An angle is a measure of rotation between two rays with a common endpoint.
- **Types of Angles:**
  - **Acute:** between 0° and 90° | **Right:** 90° | **Obtuse:** between 90° and 180° | **Straight:** 180°
  - **Adjacent Angles:** Angles with a common side and vertex but do not overlap.
  - **Supplementary & Complementary:** Supplementary ∠ sum: 180°; Complementary ∠ sum: 90°
    - **S**upplementary: **S**traight Line • **Co**mplementary: **Co**rner
  - **Linear Pair:** Two angles that are supplementary and adjacent.
  - **Vertical Angles:** Congruent angles on opposite sides of an intersection point.



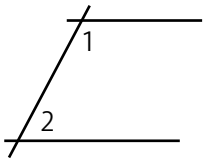
- **Parallel Lines & Transversals:** When a **transversal** crosses parallel lines, all resulting angle pairs are either congruent like vertical angles (**alternate interior**, **alternate exterior**, **corresponding**) or supplementary like a linear pair (**same-side interior**, **same-side exterior**).
  - corresponding: F →  • alternate interior: Z →  • same-side interior: C → 
  - **Parallel Lines:** equal slopes → symbol: ||
  - **Perpendicular Lines:** opposite reciprocal slopes → symbol: ⊥
  - **Perpendicular bisector:** A line, segment or ray that crosses a segment's midpoint at a 90° ∠.

86. In each figure, are the numbered angles equal or supplementary?

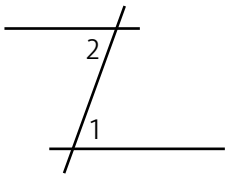
a.



b.

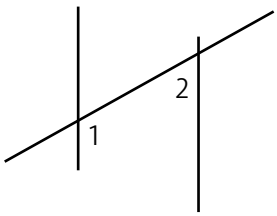


c.

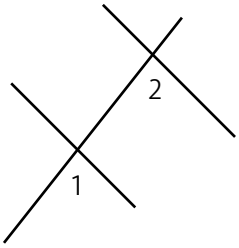


87. Are the numbered angles part of an F, C or Z?

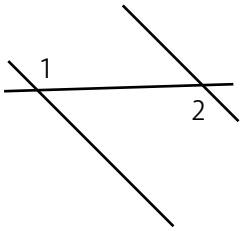
a.



b.



c.

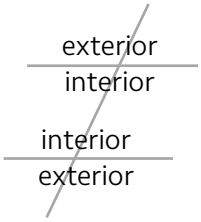


88. In the previous scenario, is each angle pair equal or supplementary?

When a transversal crosses parallel lines, angles are classified by their position.

Angles between parallel lines are **interior**.

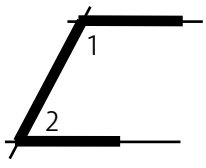
Angles outside parallel lines are **exterior**.



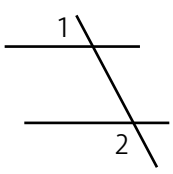
Angle pairs are also classified by their position relative to the transversal.

Alternate angles are on different sides of a transversal.

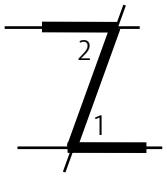
Same-side angles are on the same side of a transversal.



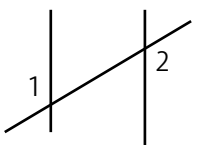
same-side interior
form a "C" → ⊏



same-side exterior

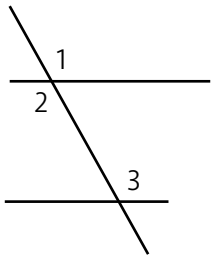


alternate interior
form a "Z" → Z

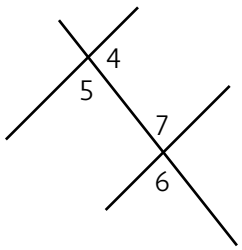


alternate exterior

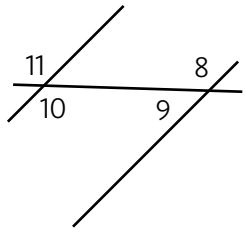
89. In each figure, name the pair of alternate interior angles. Hint: they form a "Z."

a. 

∠_____ and ∠_____

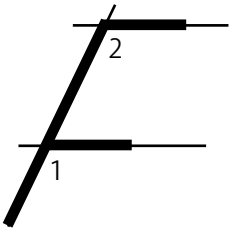
b. 

∠_____ and ∠_____

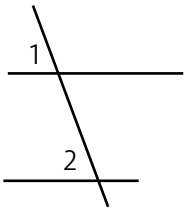
c. 

∠_____ and ∠_____

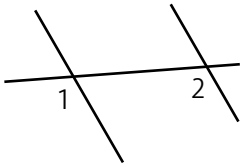
Corresponding angles have the same position at their respective intersection points.
Notice how they form an "F." → F



corresponding
(both are lower-right)

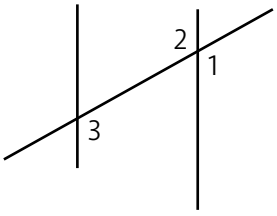


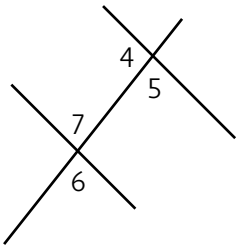
corresponding
(both are upper-left)

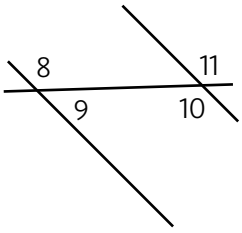


corresponding
(both are lower-left)

90. In each figure, name the pair of corresponding angles. Hint: they form an "F."

a. 

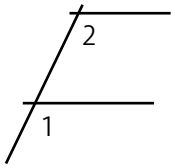
b. 

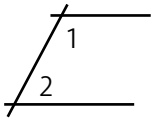
c. 

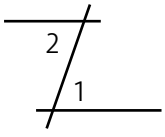
91. Which letter is formed by each angle pair, F, C or Z?

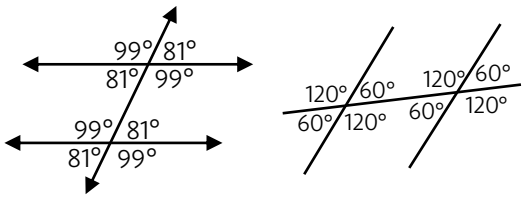
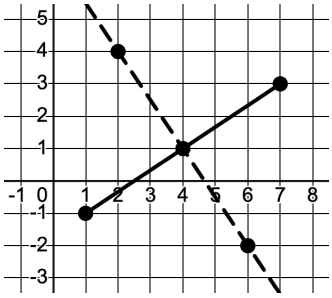
a. alternate interior b. same-side interior c. corresponding

92. What angle pair is shown in each figure?

a. 

b. 

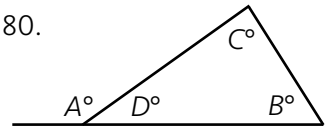
c. 

	only valid solution is $x = 5$. $m\angle EKH = 5^2 = 25°$ $m\angle HKL = 25°$
77.	a. $m\angle 1 = 21°$ $m\angle HFL = 180 - 42 = 138°$ b. $m\angle 1 + m\angle 2 + m\angle HFL = 180°$ $\rightarrow x + 12 + x + 12 + 3x + 6 = 180$ $\rightarrow 5x + 30 = 180 \rightarrow x = 30$ $\rightarrow m\angle 1 = m\angle 2 = 42° \rightarrow m\angle DFH = 84°$
78.	$\frac{1}{2}(4x + 7) \rightarrow 2x + 3.5$
79.	a. $m\angle FAC = 180° - m\angle 4 = 154°$ $m\angle 1 = m\angle 4 = 26°$ $m\angle 1 + m\angle 2 + m\angle 3 = 90°$ $m\angle 2 + m\angle 3 = 90° - 26° = 64°$ $m\angle 2 = m\angle 3 = 32°$ b. $x + 5 + 2x - 5 + 2x - 5 = 90$ $5x - 5 = 90 \rightarrow x = 19$ and $m\angle 2 = 33°$
80.	a. $\angle 1 \cong \angle 3 \cong \angle 5 \cong \angle 7$ b. $\angle 2 \cong \angle 4 \cong \angle 6 \cong \angle 8$ c. $m\angle 2 = m\angle 4 = m\angle 8 = m\angle 6 = 70°$ $m\angle 1 = m\angle 3 = m\angle 5 = m\angle 7 = 110°$ d. $180° - 68° \rightarrow 112°$
81.	a. b. 
82.	a. $m\angle 1 = 115°$. There is not enough information to find $m\angle 2$ because the figure does not show parallel lines. b. $m\angle 5 = 85°$; $m\angle 4 = 180 - 85 = 95°$. There is not enough information to find $m\angle 3$ because the lines are not parallel.
83.	$m\angle 1 = 135°$; $m\angle 2 = 45°$ $m\angle 3 = 109°$; $m\angle 4 = 71°$
84.	a. 125° b. 110° c. 64°
85.	a. F b. C c. Z
86.	a. equal b. supplementary c. equal
87.	a. C b. F c. Z
88.	a. supplementary b. equal c. equal
89.	a. $\angle 2, 3$ b. $\angle 5, 7$ c. $\angle 10, 8$
90.	a. $\angle 3, 1$ b. $\angle 5, 6$ c. $\angle 8, 11$
91.	a. Z b. C c. F
92.	a. corresponding b. same-side interior c. alternate interior
93.	a. equal b. supplementary c. equal
94.	a. alternate interior b. corresponding c. same-side interior d. same-side interior
95.	a. alternate interior b. alternate exterior c. corresponding d. same-side exterior e. corresponding f. no special name
96.	a. 50° b. 136° c. 122°
97.	solve: $x - 10 = 105 \rightarrow x = 115$ solve: $3y = 75 \rightarrow y = 25$
98.	solve: $12x - 8 = 112 \rightarrow x = 10$ solve: $5y + 2 = 112 \rightarrow y = 22$
99.	$m\angle 1 = 52°$; $180° - 115° = 65° \rightarrow m\angle 3 = 65°$ Angles 1, 2 and 3 are supplementary. $m\angle 2 = 63°$
100.	solve: $10x + 12 = 13x - 18 \rightarrow x = 10$ Since $x = 10$, then $13x - 18$ is 112. solve: $7y - 2 = 68 \rightarrow y = 10$
101.	Solve a system of equations using either substitution or elimination. $x - 2y = 40$ and $2x + y = 140$ $x = 64, y = 12$
102.	solve: $6y + 3 = 75 \rightarrow y = 12$ solve: $2x - 7 = 75 \rightarrow x = 41$
103.	There are 3 pairs of vertical angles. solve: $x - 4 = 3x - 60 \rightarrow x = 28$ The 2 smallest angles are 24°. solve: $48 + y = 60 - 2y \rightarrow y = 4$ The angles $(48 + y)°$ and $(60 - 2y)°$ are 52°. The 2 obtuse angles are both 104°.
104.	90° angles
105.	a. opposite reciprocal b. $-\frac{5}{3}$
106.	 Point-Slope Form: $y - y_1 = m(x - x_1)$ $y - 1 = -\frac{3}{2}(x - 4)$ or $y + 2 = -\frac{3}{2}(x - 6)$ or $y - 4 = -\frac{3}{2}(x - 2)$
107.	The segment's slope is $\frac{1}{3}$ $y - 7 = -3(x - 2)$

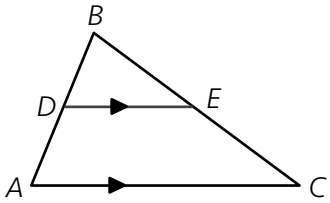
Unit 2 Summary

The key terms and concepts in this Unit:

- **Types of Triangles:**
  - Triangles are classified by **angles** or **sides**
    - **Acute:** 3 acute ∠s • **Right:** 1 right ∠ • **Obtuse:** 1 obtuse ∠ • **Equiangular:** 3 ≅ 60° ∠s
    - **Equilateral:** 3 ≅ sides • **Isosceles:** 2 ≅ sides & angles • **Scalene:** 3 different sides
- **Triangle Sum Theorem:** The sum of the interior angles of any triangle is 180°: $B + C + D = 180$.
  - **Exterior angle:** An angle formed by a polygon's side and the extension of an adjacent side.
  - An interior ∠ and its adjacent exterior ∠ form a **linear pair**: $D + A = 180$.
  - In a triangle, an exterior angle equals the sum of the triangle's 2 non-adjacent interior angles: $A = B + C$.

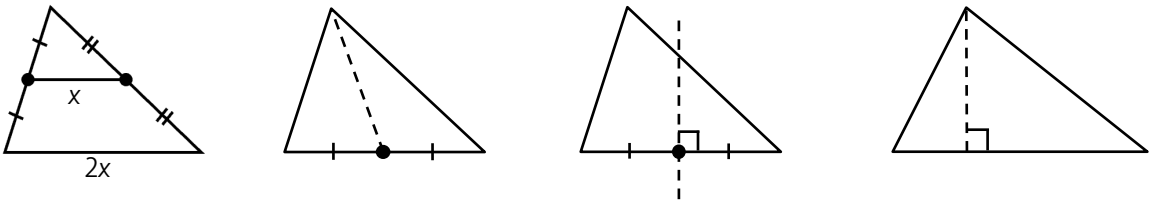


- **Similarity (~):** Similar figures have congruent corresponding angles and proportional side lengths. In the figure, $\triangle BDE \sim \triangle BAC$.
  - A proportion is two equal fractions: $\frac{a}{b} = \frac{x}{y}$
  - Parallel lines split a triangle's sides into proportional segments. In the figure, $\frac{BD}{DA} = \frac{BE}{EC}$.

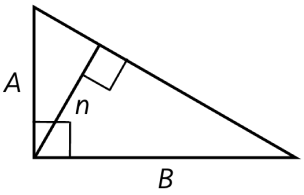


- **Segments in Triangles:**

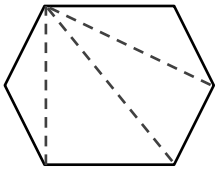
Midsegment **Median** **Perpendicular Bisector** **Altitude**



- **Geometric Mean:** The geometric mean of A and B is the number, n, that is between A and B and makes $\frac{A}{n} = \frac{n}{B}$.
  - In the figure, the length of the altitude, n, is the geometric mean of A and B.



- **Polygon:** a closed, flat shape made with straight line segments.
  - A polygon is **regular** if all sides and angles are congruent.
  - A **diagonal** is a line segment connecting non-adjacent vertices in a polygon.
  - The **sum of the interior angles** of an n-sided polygon is $(\boldsymbol{n-2}) \cdot \mathbf{180}°$.
  - A polygon's **exterior angles sum** is always **360**°.



Similar Right Triangles & The Geometric Mean

62. The **arithmetic mean** is the average of a set of numbers. If there are only 2 numbers, the arithmetic mean is exactly halfway between them. Identify the arithmetic mean of each pair.

a. 4 and 6 b. 10 and 15 c. -7 and 47 d. $\frac{1}{3}$ and $\frac{7}{5}$

63. The arithmetic mean of A and B is $\frac{A+B}{2}$. The **geometric mean** of A and B is the number, n, that is between A and B and makes $\frac{A}{n}=\frac{n}{B}$. For example, the geometric mean of 2 and 8 is 4, since $\frac{2}{4}=\frac{4}{8}$. Though $n=-4$ also makes $\frac{2}{n}=\frac{n}{8}$, it is not between 2 and 8.

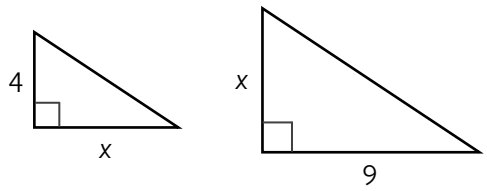
a. What is n if $\frac{3}{n}=\frac{n}{12}$? b. What is the geometric mean of 4 and 16?

64. What is the geometric mean of each pair?

a. 4 and 25 b. 4 and 6 c. 6 and 15 d. $\frac{1}{3}$ and $\frac{3}{4}$

65. The geometric mean occurs in geometry. Consider the pair of similar right triangles.

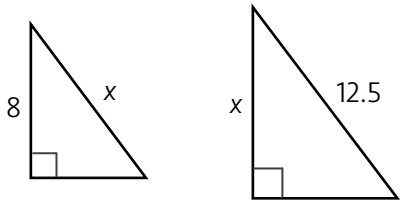
a. Use a proportion to find the value of x.



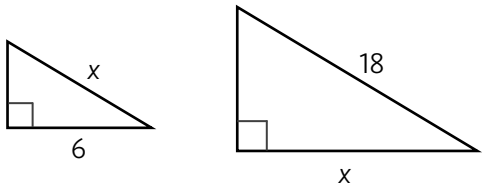
b. The geometric mean of 4 and 9 is ____.

66. Consider each pair of similar right triangles. Use a proportion to find the value of x.

a.

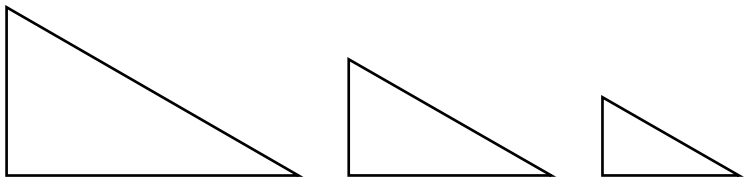


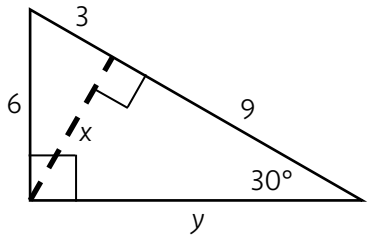
b.



When you draw the altitude to the hypotenuse of a right triangle (the dashed line), it forms 3 triangles with a special relationship between their side lengths.

67. The dashed altitude creates a smaller, medium and larger triangle. They are drawn separately below.

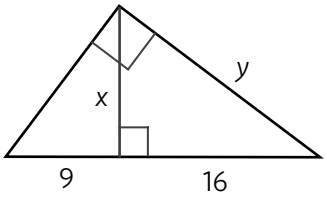


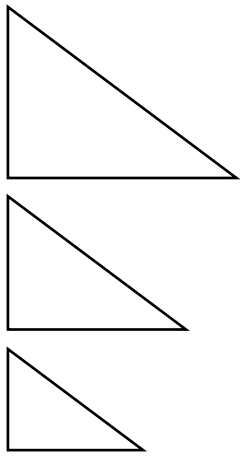


a. Use the 30° angle in the figure to identify the measure of all 9 angles in the 3 triangles. What do these angle measures reveal about these 3 triangles?

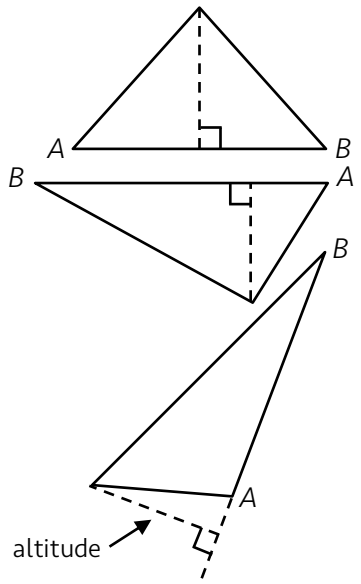
b. Label the side lengths of the 3 triangles drawn for you. Use your observation in part a. to identify the values of x and y.

68. The figure contains 3 similar triangles, a smaller, medium and larger triangle. It is easier to compare the corresponding sides if you draw each triangle separately. Label the side lengths of the 3 triangles that are already drawn for you. Use proportions to find x and y.





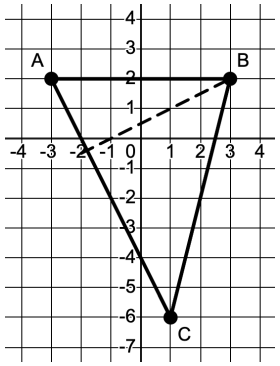
58.



59. a. median b. $\overline{AD}$ is a median, altitude and perpendicular bisector. These are all the same segment in an isoscelese triangle.

60.

a.



b. contains point B

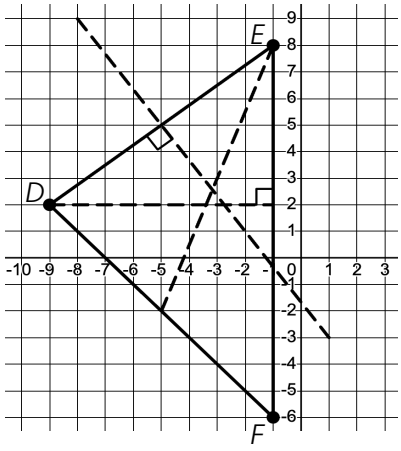
slope is ⊥ to $\overline{AC}$: $\frac{1}{2}$

equation: $y-2=\frac{1}{2}(x-3)$

c. Find the point where the altitude intersects $\overline{AC}$ using the altitude's equation and the equation of $\overline{AC}$. Solve that system.

The altitude's length is the distance between that point and point B.

61.



a. midpoint: $(-5,-2)$ slope: $\frac{5}{2}$

equation: $y+2=\frac{5}{2}(x+5)$

b. midpoint: $(-5,5)$ slope: $-\frac{4}{3}$

equation: $y-5=-\frac{4}{3}(x+5)$

c. altitude is horizontal $\rightarrow y=2$

d. Find distance between each vertex

$DE=\sqrt{(-9+1)^2+(2-8)^2}=10$

$EF=14$ $DF=\sqrt{128}=8\sqrt{2}$

Perimeter: $24+8\sqrt{2}$

e. Half of the triangle's perimeter: $12+4\sqrt{2}$

62. a. 5 b. 12.5 c. $\frac{-7+47}{2}\rightarrow 20$

d. $\frac{5}{15}+\frac{21}{15}\rightarrow\frac{26}{15}\cdot\frac{1}{2}\rightarrow\frac{13}{15}$

63. a. $n^2=36\rightarrow n=\pm 6$

b. $\frac{4}{n}=\frac{n}{16}\rightarrow n^2=64\rightarrow n=\pm 8$

The geometric mean is between 4 and 16, so 8 is the geometric mean. $\frac{4}{8}=\frac{8}{16}$

64. a. 10 b. $2\sqrt{6}$ c. $3\sqrt{10}$ d. $\frac{1}{2}$

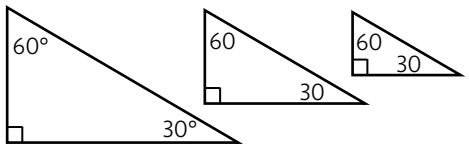
65. a. $\frac{4}{x}=\frac{x}{9}\rightarrow x^2=36\rightarrow x=6$

b. 6, because $\frac{4}{6}=\frac{6}{9}$

66. a. $\frac{8}{x}=\frac{x}{12.5}\rightarrow x=10$

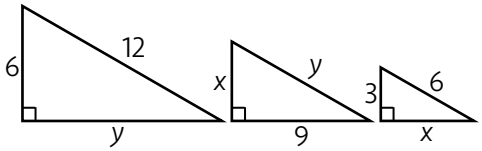
b. $\frac{6}{x}=\frac{x}{18}\rightarrow x=6\sqrt{3}$

67.



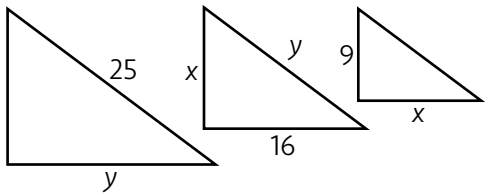
a. Since all 3 of their angles are conguent, they are similar triangles.

b.



$\frac{x}{3}=\frac{9}{x}\rightarrow x=3\sqrt{3}$ $\frac{y}{9}=\frac{12}{y}\rightarrow y=6\sqrt{3}$

68.



$\frac{x}{9}=\frac{16}{x}\rightarrow x=12$ $\frac{y}{16}=\frac{25}{y}\rightarrow y=20$

69. a. $\frac{4.5}{x}=\frac{x}{8}\rightarrow x=6$

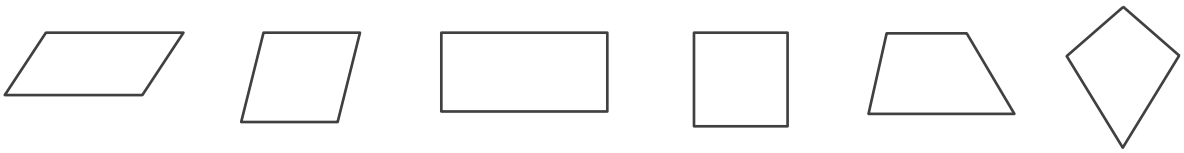
b. $\frac{4}{x}=\frac{x}{10}\rightarrow x=2\sqrt{10}$

Unit 3 Summary

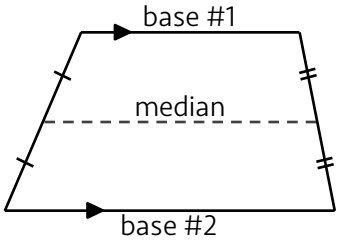
The key terms and concepts in this Unit:

- **6 Types of Quadrilaterals:**

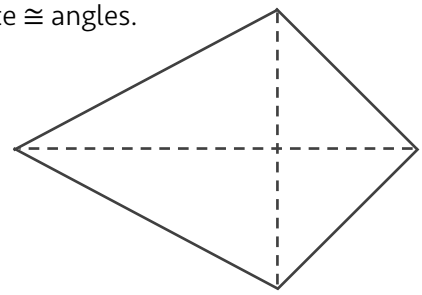
parallelogram rhombus rectangle square trapezoid kite



- **Parallelograms:** Quadrilaterals with two pairs of opposite sides parallel.
    - Opposite sides are ≅ and opposite angles are ≅.
    - Consecutive angles are supplementary.
    - Diagonals bisect each other.
- **Special Parallelograms:**
    - **Rhombus:** Four ≅ sides; diagonals are ⊥.
    - **Rectangle:** Four right angles and ≅ diagonals.
    - **Square:** A regular quadrilateral that is both a rhombus and a rectangle.
- **Trapezoids & Kites:**
    - **Trapezoid:** A quadrilateral with exactly one pair of parallel bases and 2 non-parallel legs.
        - An **isosceles trapezoid** has 2 ≅ legs and 2 ≅ angles.
        - A **median** connects midpoints of the non-parallel sides.
        - The median's length is the average of the bases.



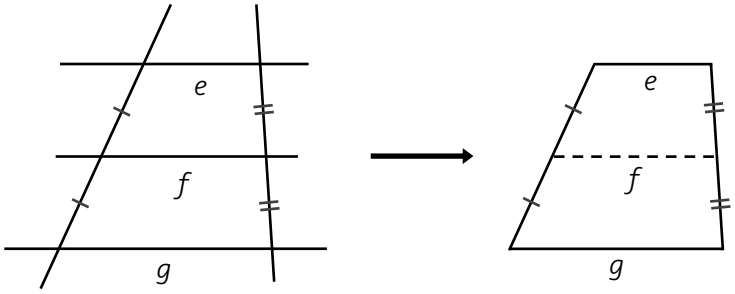
    - **Kite:** A quadrilateral with two pairs of ≅ adjacent sides.
        - One pair of opposite ≅ angles.
        - Diagonals are ⊥.



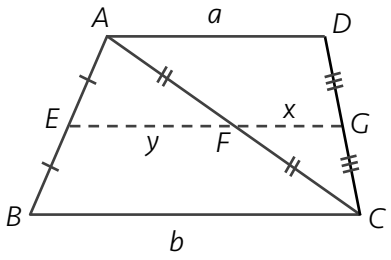
The Median of a Trapezoid

56. Parallel lines split 2 transversals into proportional segments. They also form trapezoids. In the trapezoid shown, a dashed segment connects the midpoints of its non-parallel sides. This midpoint-connecting segment is called a **median**.

Make a guess. If $e = 6$ and $g = 10$, the length of f is _____.



A trapezoid's **median** length is the average of its base lengths.
To see this, split a trapezoid into 2 triangles, as shown.



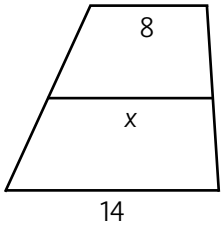
EF and FG are midsegments, so $x = \frac{a}{2}$ and $y = \frac{b}{2}$

$EG = x + y = \frac{a}{2} + \frac{b}{2} = \frac{a+b}{2}$

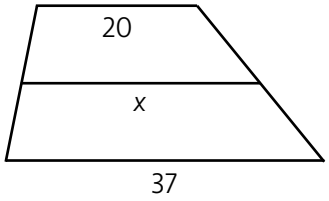
EG, the median, is the average of a and b $\rightarrow$ $\frac{a+b}{2}$

57. The median of the trapezoid is shown. What is its length, x?

a.

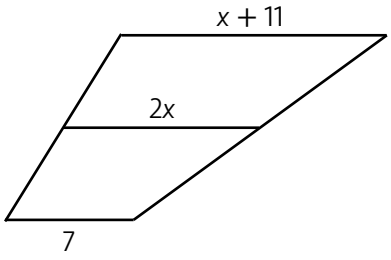


b.

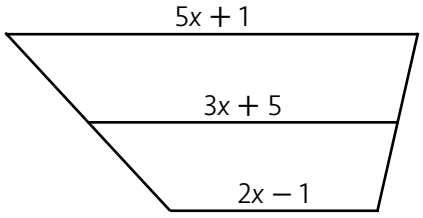


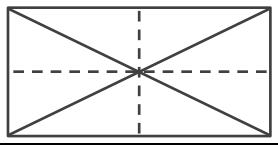
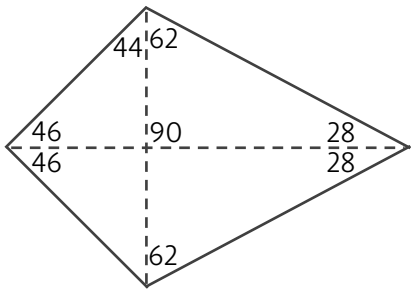
58. What is the length of each trapezoid's median?

a.



b.

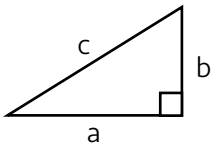


38.	a. 29° b. 70° c. 62° d. $(90 - x)°$
39.	a. $m\angle 1 = 55°$; $m\angle 2 = 35°$ b. $m\angle 2 = 27°$; $m\angle 1 = 63°$
40.	a. The diagonals do not form 90° angles. b. The shorter diagonal does not split the longer diagonal into 2 congruent segments.
41.	a. Rhombus diagonals bisect each other. If $WY = 12$, then $WO = 6$. If $XZ = 16$, then $XO = 8$. WOX is a right triangle because rhombus diagonals are perpendicular. Use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to find WX. b. $6^2 + 8^2 = (WX)^2 \rightarrow WX = 10$
42.	Each side of the rhombus is one-fourth of the perimeter $\rightarrow$ 15. The diagonals make 4 congruent right triangles. The shorter leg of each right triangle is 9 and the hypotenuse is 15. $9^2 + (YM)^2 = 15^2 \rightarrow YM = 12$
43.	rectangle
44.	The diagonals are congruent.
45.	a. $\sqrt{7^2 + 1^2} \rightarrow \sqrt{50} \rightarrow 5\sqrt{2}$ b. $\sqrt{5^2 + 5^2} \rightarrow \sqrt{50} \rightarrow 5\sqrt{2}$ c. A rectangle has congruent diagonals.
46.	a. Solve: $7x - 9 = 4x + 6 \rightarrow x = 5$ $AC = 7x - 9 = 7(5) - 9 \rightarrow AC = 26$ units b. To find the perimeter, find the length of BC using the Pythagorean Theorem. $10^2 + (BC)^2 = 26^2 \rightarrow BC = 24$ The perimeter is $2 \cdot 10 + 2 \cdot 24 \rightarrow 68$ units.
47.	The diagonals create two pairs of congruent isosceles triangles. Unless the rectangle is a square, 2 of the triangles are acute isosceles and 2 are obtuse isosceles.
48.	a. $TH = 12$, $LT = 12$, $PX = 24$ b. $TL = 22$, $PX = 44$
49.	a. $m\angle TPH = 65°$ $m\angle LTX = 130°$ b. $m\angle XLT = 38°$ $m\angle LTX = 71°$
50.	4 cans. Each triangle has the same area. The dashed lines below show that the 4 isosceles triangles formed by the diagonals contain the same 2 congruent right triangles. 
51.	A rectangle has congruent diagonals. $4x + 13 = 9x - 47 \rightarrow x = 12$
52.	a. $m\angle 1 = 180° - 75° \rightarrow 105°$ b. 112° c. 65°
53.	a. $m\angle 3 = 72°$ $m\angle 1 = m\angle 2 = 108°$ b. $m\angle 3 = 135°$ $m\angle 1 = m\angle 2 = 45°$
54.	a. $3x = 63 \rightarrow x = 21$ b. $18x - 3 = 6x + 1 \rightarrow x = \frac{1}{3}$
55.	The larger triangle has a base of 9. $A = \frac{1}{2}bh \rightarrow 27 = \frac{1}{2}(9)h \rightarrow 54 = 9h \rightarrow 6 = h$ The height of the smaller triangle is also 6. Area of smaller triangle: $42 - 27 \rightarrow 15$ $A = \frac{1}{2}bh \rightarrow 15 = \frac{1}{2}x(6) \rightarrow 30 = 6x \rightarrow 5 = x$
56.	average of 6 and 10 $\rightarrow$ 8
57.	a. 11 b. $\frac{20 + 37}{2} \rightarrow 28.5$
58.	a. $\frac{x + 11 + 7}{2} = 2x \rightarrow x + 18 = 4x \rightarrow x = 6$ b. $\frac{5x + 1 + 2x - 1}{2} = 3x + 5$ $\rightarrow 7x = 6x + 10 \rightarrow x = 10$
59.	a. $AB = \sqrt{9^2 + 3^2} \rightarrow \sqrt{90} \rightarrow 3\sqrt{10}$ $CD = \sqrt{3^2 + 1^2} \rightarrow \sqrt{10}$ b. The median connects the midpoints of the nonparallel sides, AD and BC. AD midpoint: $(-4, -2)$ BC midpoint: $(2, 0)$ median length: $= \sqrt{6^2 + 2^2} \rightarrow \sqrt{40} \rightarrow 2\sqrt{10}$ $2\sqrt{10}$ *is* the average of $\sqrt{10}$ and $3\sqrt{10}$ $\rightarrow$ median length is average of base lengths
60.	2 pairs of adjacent, congruent sides 1 pair of opposite congruent angles perpendicular diagonals 1 diagonal is bisected by the other diagonal
61.	a. $m\angle 2 = 93°$ $m\angle 1 = 360 - (93 + 93 + 108) \rightarrow m\angle 1 = 66°$ b. $360 - (89 + 56) \rightarrow 215 \div 2 = 107.5$ $m\angle 1 = m\angle 2 = 107.5°$
62.	a. $m\angle 3 = 90°$ $m\angle 2 = 90 - 35 = 55°$ $m\angle 1 = 90 - 36 = 54°$ b. $m\angle 1 = 90 - 44 = 46°$ $m\angle 2 = 62°$ $m\angle 3 = 90 - 62 = 28°$ 
63.	$2(2x + 30) + (3x - 10) + 107 = 360$ $7x + 157 = 360 \rightarrow x = 29$ $m\angle A = 2x + 30 = 2(29) + 30 = 88°$

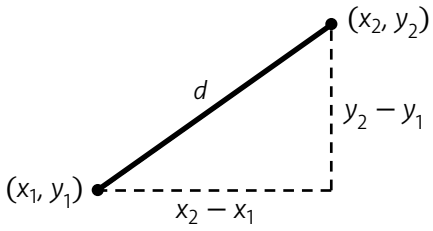
Unit 4 Summary

The key terms and concepts in this Unit:

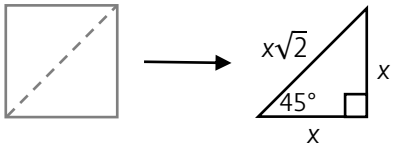
- **Pythagorean Theorem:**
  - A formula to find a missing side length in a right triangle.
    - $a^2 + b^2 = c^2$
    - "c" is the **Hypotenuse**, opposite the right angle
    - "a" and "b" are called **Legs**



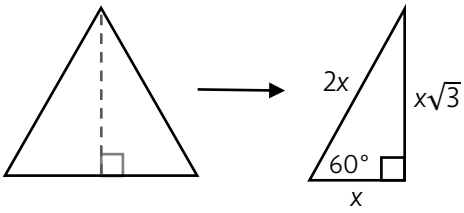
- **The Distance Formula:**
  - A formula to calculate the length between any two points.
  - It is derived from the Pythagorean Theorem.
    - $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



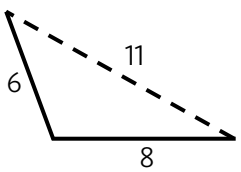
- **45°-45°-90° Triangles → $x : x : x\sqrt{2}$**
  - An isosceles right triangle; half of a square.
    - Both Legs: x
    - Hypotenuse: $x\sqrt{2}$



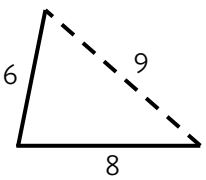
- **30°-60°-90° Triangles → $x : x\sqrt{3} : 2x$**
  - Half of an equilateral triangle.
    - Shorter leg: x
    - Longer Leg: $x\sqrt{3}$
    - Hypotenuse: $2x$



- **Using the Pythagorean Theorem to Classify a Triangle:**
  - **Triangle Inequality Theorem:** If a triangle has side lengths a, b and c, then...
    - Any 2 side lengths of a △ have a sum greater than the 3rd side length → $a + b > c$
    - Given 2 side lengths, a and b, the third side, c, must be in this range → $|a - b| < c < a + b$
  - **Is a triangle acute, right or obtuse?**
    - **Acute:** $c^2 < a^2 + b^2$
    - **Right:** $c^2 = a^2 + b^2$
    - **Obtuse:** $c^2 > a^2 + b^2$



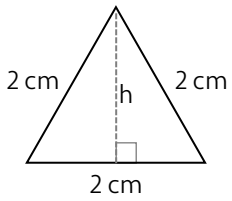
$c^2 > a^2 + b^2$



$c^2 < a^2 + b^2$

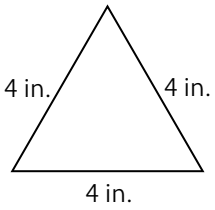
A Special Right Triangle in an Equilateral Triangle

36. Another special right triangle can be found in an equilateral triangle. Use the Pythagorean Theorem to find the height of the triangle. Write the height in simplified radical form.

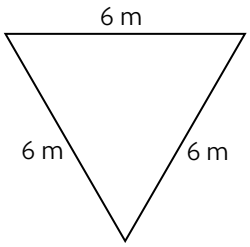


37. Use the Pythagorean Theorem to find the height of each triangle, in simplified radical form.

a.

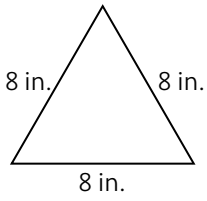


b.

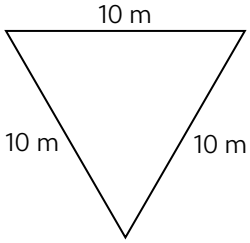


38. Find the height of each triangle shown below and try to find a relationship between the height and the triangle's side lengths.

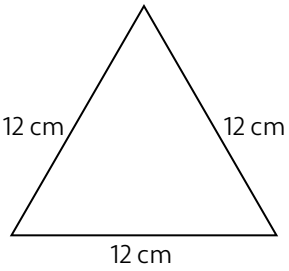
a.



b.



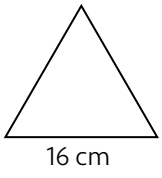
c.



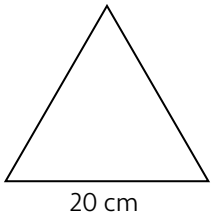
Try to find a pattern in the previous scenarios by comparing the height and the side lengths of each equilateral triangle. If you see this pattern, the next question can be answered without using the Pythagorean Theorem.

39. What is the height of each equilateral triangle shown?

a.



b.



Classifying Triangles with the Pythagorean Theorem

88. Do 3 connected segments always form a triangle? Try to draw a triangle with the 3 segments shown. What type of triangle is formed?

3

3

3

89. Try to draw a triangle with the 3 segments shown. What type of triangle is formed?

5

4

3

90. What type of triangle is formed when you connect each group of 3 segments shown?

a.

5

1

1

b.

6

4

2

Triangle Inequality Theorem

If a triangle has side lengths a, b and c, then any 2 side lengths have a sum greater than the 3rd side.

$$a + b > c$$

If only 2 side lengths are known, a and b, then the third side, c, must be in this range:

$$|a - b| < c < a + b$$

91. Can you form a triangle with the 3 segment lengths shown?

a. 8, 12, 21 b. 5, 14, 19 c. 9, 15, 12

79. height: $4\sqrt{3}$ area: $16\sqrt{3}$ cm^2

80. base: 5 height: $\frac{5\sqrt{15}}{2}$ inches ($\approx$9.68 inches)
area: $\frac{25\sqrt{15}}{4}$ inches^2 ($\approx$24.2 inches^2)

81. both legs: $\frac{1}{2}$ Area: $\frac{1}{8}$ units^2

82. A: 5; base: 6; height: 4; Area = 12 units^2

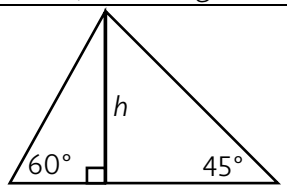
83. square side length: $4\sqrt{3}$
triangle base length: $\frac{16\sqrt{3}}{3}$ in.

84. triangle perimeter: $3 \cdot 6 \rightarrow 18$
square side length: $18 \div 4 \rightarrow \frac{9}{2}$
square's diagonal: $\frac{9\sqrt{2}}{2}$ cm or $4.5\sqrt{2}$ cm

85. solve: $y^2 + 15^2 = 17^2 \rightarrow y = \pm 8$
triangle's side length is positive: $y = 8$

86. solve: $x^2 + (x-7)^2 = (x+1)^2 \rightarrow x = 12$ or 4
solution: $x = 12$
If x is 4, the triangle side length is negative.

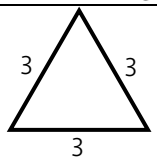
87.



In the 30-60-90 triangle:
$h = x\sqrt{3}$
In the 45-45-90 triangle:
$h = 100 - x$

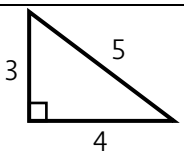
Solve $100 - x = x\sqrt{3}$.
$\rightarrow 100 = x\sqrt{3} + x \rightarrow 100 = x(\sqrt{3}+1)$
$\rightarrow x = \frac{100}{1+\sqrt{3}} \approx 36.6$; $h = \frac{100\sqrt{3}}{1+\sqrt{3}} \approx 63.4$
The kite's height is $63.4 + 5 \rightarrow 68.4$ ft

88.



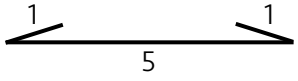
equilateral, equiangular △

89.

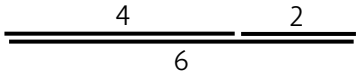


scalene, right △

90. a. No triangle can be made. The sum of the 2 shorter segments must be more than 10.



b. No triangle can be made. The sum of the 2 shorter segments must be more than 6. These 3 segments would form a flat triangle with no height.



91. The sum of the 2 smaller sides must be greater than the largest side.
a. $8 + 12 = 20 < 21 \rightarrow$ not a triangle
b. $5 + 14 = 19 < 21 \rightarrow$ not a triangle
c. yes $\rightarrow 9 + 12 = 21 > 15$

92. $12 - 9 < c < 12 + 9 \rightarrow 3 < c < 21$
The 3rd side must be between 3 and 21.

93. a. $10 - 3 < c < 10 + 3 \rightarrow 7 < c < 13$
The 3rd side must be between 7 and 13.
b. $7 - 7 < c < 7 + 7 \rightarrow 0 < c < 14$
The 3rd side must be between 0 and 14.
c. The 3rd side must be between 1 and 23.

94. First, confirm the sides form a triangle:
$a + b > c$ if c is the longest side
a. $7 + 24 > 25 \rightarrow$ they form a triangle
c^2: $25^2 = 625$ $a^2 + b^2$: $7^2 + 24^2 = 625$
$c^2 = a^2 + b^2 \rightarrow$ right △
b. $10 + 10 > 12 \rightarrow$ they form a triangle
c^2: $12^2 = 144$ $a^2 + b^2$: $10^2 + 10^2 = 200$
$c^2 < a^2 + b^2 \rightarrow$ acute △
c. $5 + 7 < 13 \rightarrow$ not a triangle

95. First, confirm the sides form a triangle:
$a + b > c$ if c is the longest side
a. $8 + 15 > 17 \rightarrow$ they form a triangle
c^2: $17^2 = 289$ $a^2 + b^2$: $8^2 + 15^2 = 289$
$c^2 = a^2 + b^2 \rightarrow$ right △
b. $6 + 7 > 12 \rightarrow$ they form a triangle
c^2: $12^2 = 144$ $a^2 + b^2$: $6^2 + 7^2 = 85$
$c^2 > a^2 + b^2 \rightarrow$ obtuse △
c. $\sqrt{5} > 2$ $\sqrt{8} > 2$ $\sqrt{13} < 4$
$\sqrt{5} + \sqrt{8} > \sqrt{13} \rightarrow$ they form a triangle
c^2: $(\sqrt{13})^2 = 13$ $(\sqrt{5})^2 + (\sqrt{8})^2 = 13$
$c^2 = a^2 + b^2 \rightarrow$ right △

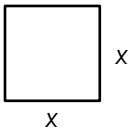
96. Note: The longest side could be either c or 12 and given 2 sides of 9 and 12 $\rightarrow 3 < c < 21$
a. If c is the longest side:
$c^2 = 9^2 + 12^2 \rightarrow c = \sqrt{225} = 15$
If 12 is the longest side:
$12^2 = 9^2 + c^2 \rightarrow c = \sqrt{63} = 3\sqrt{7}$
b. If c is the longest side:
$c^2 < 9^2 + 12^2 \rightarrow 12 < c < 15$
If 12 is the longest side:
$12^2 < 9^2 + c^2 \rightarrow 3\sqrt{7} < c < 12$
If $c = 12$, the △ is still acute so the range of possible c-values is this: $3\sqrt{7} < c < 15$.
c. If c is the longest side:
$c^2 > 9^2 + 12^2 \rightarrow 15 < c < 21$
If 12 is the longest side:
$12^2 > 9^2 + c^2 \rightarrow 3 < c < 3\sqrt{7}$

Unit 5 Summary

The key terms and concepts in this Unit:

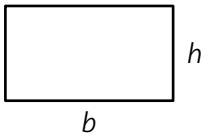
- **Area:** "The number of squares that can fit inside."
- **Basic Formulas:**
  - **Square**

    $A = x^2$

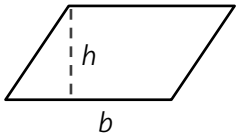
  - **Rectangle**

    $A = bh$

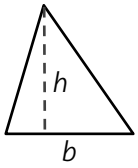
  - **Parallelogram**

    $A = bh$

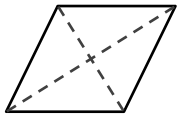
  - **Triangle**

    $A = \frac{1}{2}bh$

  - **Rhombus**

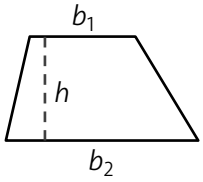
    $A = 4 \cong$ right $\triangle$s

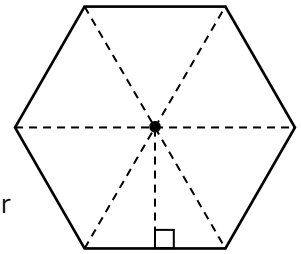
    or $A = \frac{1}{2}d_1d_2$

  - **Trapezoid**

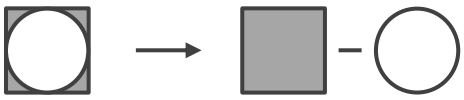
    $A = \frac{1}{2}h(b_1 + b_2)$

- **Area of a Regular Polygon:**
  - A **regular polygon** is both equilateral and equiangular.
  - Area is found by splitting the *n*-sided polygon into *n* congruent isosceles triangles.
  - **Apothem:** Segment from the center of a regular polygon perpendicular to a side; the "height" of the isosceles triangles within the polygon.

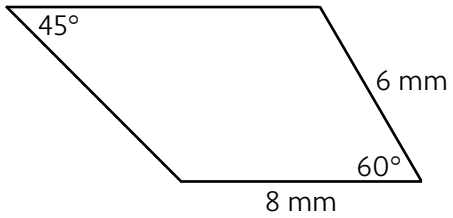


- **Area of a Shaded Region:**
  - Subtract the area of a smaller shape from a larger one (example: Square Area − Circle Area).



- **Area Ratios of Similar Figures**
  - If similar figures have a scale factor of $x : y$, then their area ratio is $x^2 : y^2$.

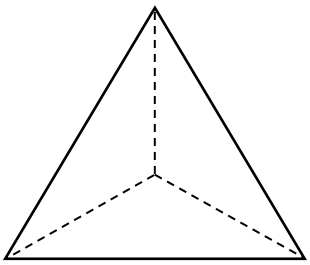
62. Calculate the trapezoid's perimeter, rounded to the nearest tenth.



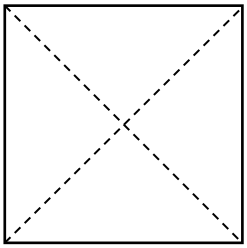
Area of a Regular Polygon

63. A polygon is a **regular polygon** if all sides are congruent. Consider each regular polygon below. A regular polygon's area is easier to find if you see the N-sided polygon as N isosceles triangles. If you draw segments from the center to each vertex, how many isosceles triangles fit inside each polygon?

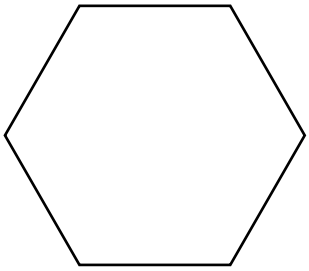
a.



b.

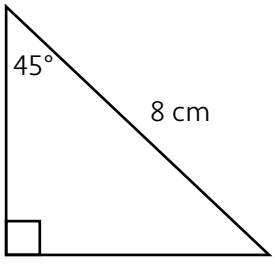


c.

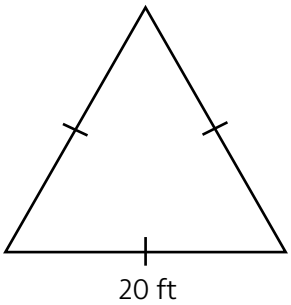


64. Since the area of a regular polygon with N sides is the sum of the areas of N isosceles triangles, first review finding the area of a triangle. Calculate the area of each triangle shown.

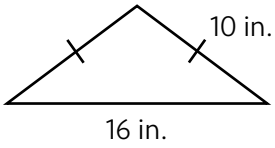
a.



b.

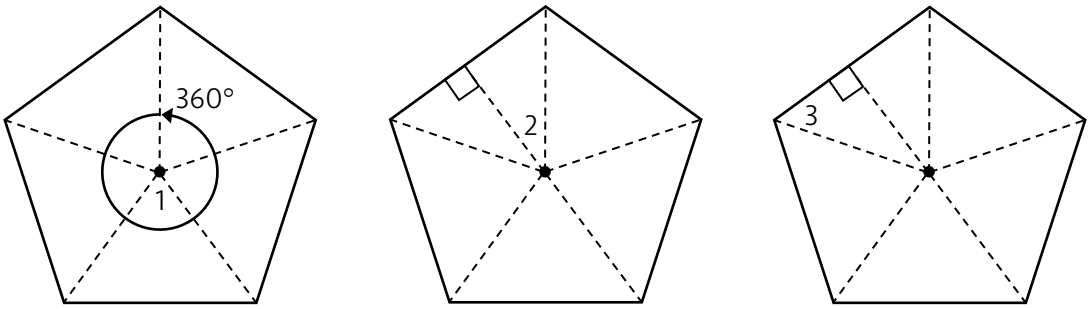


c.



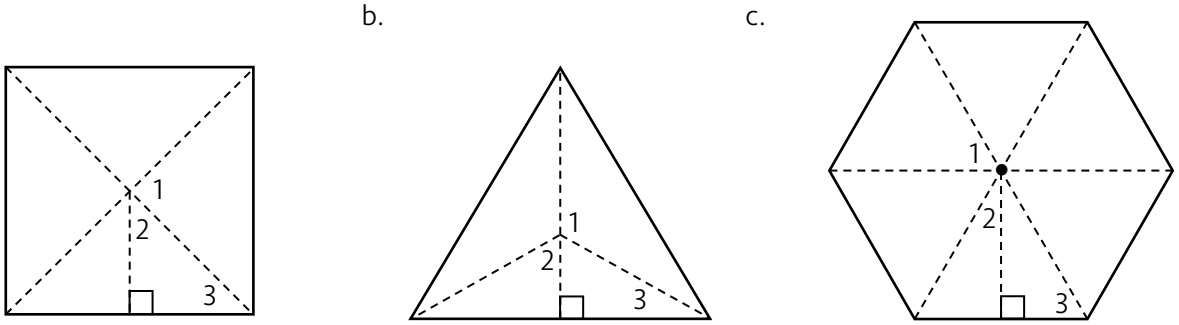
65. To find the angles in a polygon, use what you know about circles and triangles.

a. $m\angle 1 =$ b. $m\angle 2 =$ c. $m\angle 3 =$



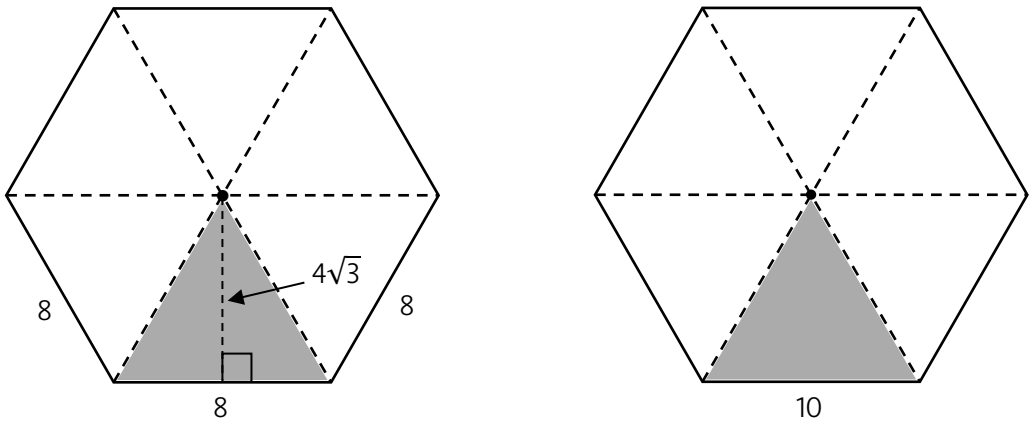
66. Find the measure of each numbered angle below.

a. b. c.



67. Find the area of the shaded region.

a. b.

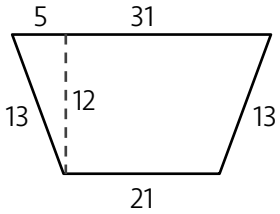


56.

a. $A = \frac{1}{2}h(B_1 + B_2)$

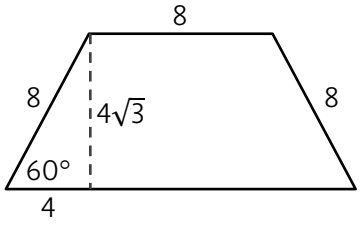
$\rightarrow \frac{1}{2} \cdot 9(16 + 20) \rightarrow 162 \text{ mm}^2$

b.



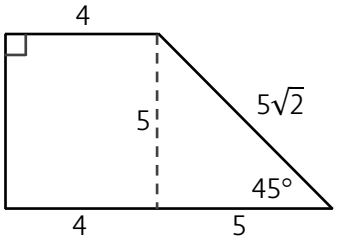
$A = \frac{1}{2} \cdot 12(31 + 21) \rightarrow 312 \text{ units}^2$

57.



a. The dashed line forms a 30-60-90 triangle. The trapezoid's height is $4\sqrt{3}$.

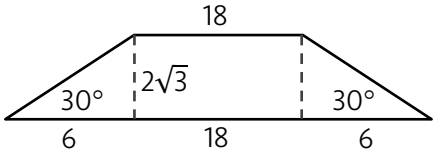
$A = \frac{1}{2} \cdot 4\sqrt{3}(16 + 8) \rightarrow 48\sqrt{3} \text{ units}^2$



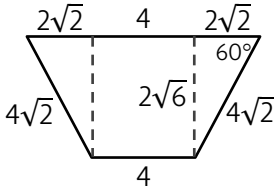
b. The dashed line forms a 45-45-90 triangle. The trapezoid's height is 5.

$A = \frac{1}{2} \cdot 5(9 + 4) \rightarrow 32.5 \text{ units}^2$

58.



$A = \frac{1}{2} \cdot 2\sqrt{3}(30 + 18) \rightarrow 48\sqrt{3} \text{ units}^2$



$A = \frac{1}{2} \cdot 2\sqrt{6}\left(4 + 4\sqrt{2} + 4\right)$

$\rightarrow \sqrt{6}\left(8 + 4\sqrt{2}\right) \rightarrow 8\sqrt{6} + 8\sqrt{3} \text{ units}^2$

59.

$40 = \frac{1}{2} \cdot h(7 + 13) \rightarrow 40 = 10h \rightarrow h = 4 \text{ in.}$

60.

$100 = \frac{1}{2} \cdot 12(10 + FE) \rightarrow 100 = 60 + 6FE$

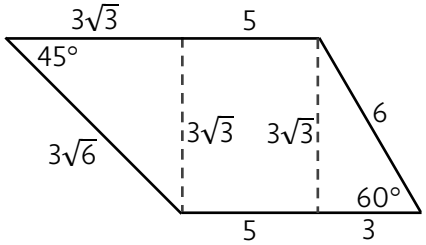
$\rightarrow h = \frac{20}{3}$ or $6\frac{2}{3}$ cm

61.

$385 = \frac{1}{2} \cdot 22(14 + B_2) \rightarrow 385 = 11(14 + B_2)$

$35 = 14 + B_2 \rightarrow B_2 = 21 \text{ m}$

62.



$P = 3\sqrt{6} + 3\sqrt{3} + 5 + 6 + 3 + 5 \approx 31.5 \text{ mm}$

63. a. 3 b. 4 c. 6

64.

a. Each leg is $4\sqrt{2}$. $A = \frac{4\sqrt{2} \cdot 4\sqrt{2}}{2} \rightarrow 16 \text{ cm}^2$

b. The height is $10\sqrt{3}$.

$A = \frac{20 \cdot 10\sqrt{3}}{2} \rightarrow 100\sqrt{3} \text{ ft}^2$

c. The height is 6. $A = \frac{16 \cdot 6}{2} \rightarrow 48 \text{ in.}^2$

65.

a. $360° \div 5 = \rightarrow 72°$

b. $72° \div 2 = \rightarrow 36°$

c. $90° - 36° = \rightarrow 54°$

66.

a. $360° \div 4 \rightarrow 90° = m\angle 1$

$90° \div 2 \rightarrow 45° = m\angle 2$

$90° - 45° \rightarrow 45° = m\angle 4$

b. $360° \div 3 \rightarrow 120° = m\angle 1$

$120° \div 2 \rightarrow 60° = m\angle 2$

$90° - 60° \rightarrow 30° = m\angle 3$

c. $360° \div 6 \rightarrow 60° = m\angle 1$

$60° \div 2 \rightarrow 30° = m\angle 2$

$90° - 30° \rightarrow 60° = m\angle 3$

67.

a. $\frac{8 \cdot 4\sqrt{3}}{2} \rightarrow 16\sqrt{3} \text{ units}^2$

b. $\frac{10 \cdot 5\sqrt{3}}{2} \rightarrow 25\sqrt{3} \text{ units}^2$

68.

a. $16\sqrt{3} \cdot 6 = 96\sqrt{3} \text{ units}^2$

b. $25\sqrt{3} \cdot 6 = 150\sqrt{3} \text{ units}^2$

69.

a. $\frac{6 \cdot 3}{2} \rightarrow 9 \text{ units}^2$

b. $\frac{6 \cdot \sqrt{3}}{2} \rightarrow 3\sqrt{3} \text{ units}^2$

70.

a. $9 \cdot 4 = 36 \text{ units}^2$

b. $3\sqrt{3} \cdot 3 = 9\sqrt{3} \text{ units}^2$

71.

a. $m\angle 1 = 360° \div 6 = 60°$

$m\angle 2 = 60° \div 2 = 30°$

$m\angle 3 = 90° - 30° = 60°$

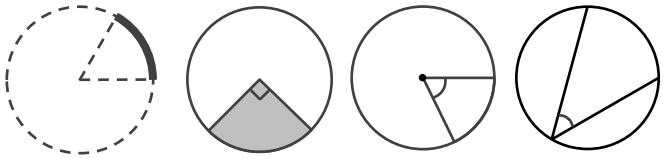
b. $x = 6$ c. $A = \frac{1}{2}\left(12 \cdot 6\sqrt{3}\right) = 36\sqrt{3} \text{ in.}^2$

d. multiply by 6 $\rightarrow 36\sqrt{3} \cdot 6 \rightarrow 216\sqrt{3} \text{ in.}^2$

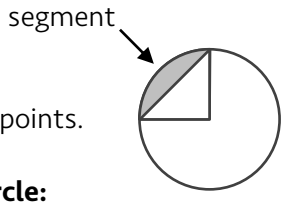
Unit 6 Summary

The key terms and concepts in this Unit:

- **Arc:** Part of a Circle • **Semicircle:** 180° • **Minor Arc:** $< 180°$ • **Major Arc:** $> 180°$
- **Central Angle:** Angle with vertex at the center: **equal** to the intercepted arc measure.
- **Inscribed Angle:** Angle with vertex on the circle: **half** the intercepted arc measure.
- **Circumference & Arc Length:**
  - **Circumference:** $C = \pi d$ or $C = 2\pi r$
  - **Arc Length, *L*:** $L = \frac{x}{360} \cdot 2\pi r$

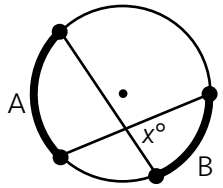


- **Area & Sectors:**
  - **Circle Area:** $A = \pi r^2$
  - **Sector Area:** Area $= \frac{x}{360} \cdot \pi r^2$



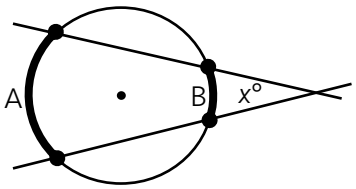
Segment: The region bounded by a chord and the arc between that chord's endpoints.

- **Angles Inside a Circle:**



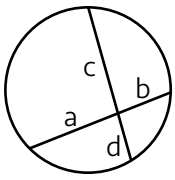
$x = \frac{A + B}{2}$

- **Angles Outside a Circle:**



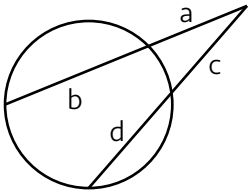
$x = \frac{A - B}{2}$

- **Intersecting Chords, Secants & Tangents:**



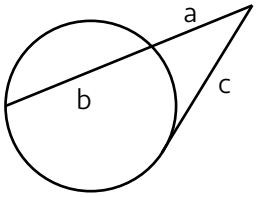
chord–chord

$ab = cd$



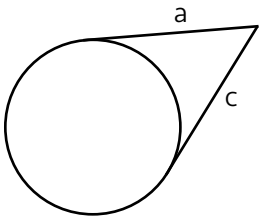
secant–secant

$a(a + b) = c(c + d)$



secant–tangent

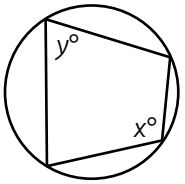
$a(a + b) = c^2$



tangent–tangent

$a = c$

- **Perpendicular Tangents:** A circle's tangent line is perpendicular to its radius.
- **Perpendicular Chords:** A radius is a chord's perpendicular bisector.
- **Inscribed Quadrilaterals:** Opposite angles have a sum of 180°.

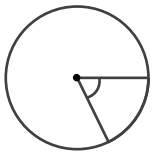


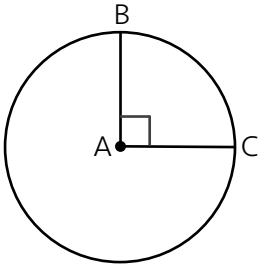
$x + y = 180$

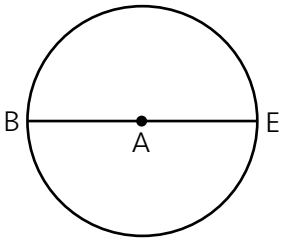
- **Equation of a Circle:** $(x - h)^2 + (y - k)^2 = r^2$
  - center: (h, k) • radius: r

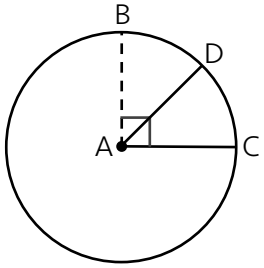
Central Angles & Inscribed Angles

4. A **central angle** has its vertex at the circle's center. The angle's "arms" are radii of the circle. There is a simple relationship between a central angle's size and the arc it intercepts. Use the 3 central angles shown to help you discover this relationship. Try to fill in each blank.









a. $m\angle BAC =$ _____ b. $m\angle BAE =$ _____ c. $m\angle DAC =$ _____

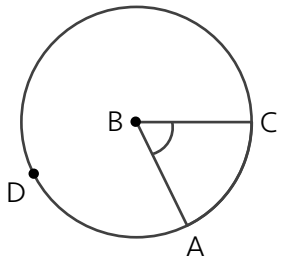
$m\overset{\frown}{BC} =$ _____ $m\overset{\frown}{BE} =$ _____ $m\overset{\frown}{DC} =$ _____

d. What do you notice when you compare each central angle to its intercepted arc?

5. Using the previous scenario, try to fill in each blank.

a. If $m\angle ABC = 61°$, then $m\overset{\frown}{AC} =$ _____.

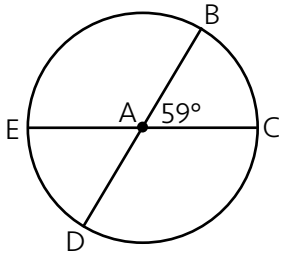
b. $m\overset{\frown}{ADC} =$ _____.



A **central angle** equals the measure of the circle's intercepted arc.

6. Identify each measure.

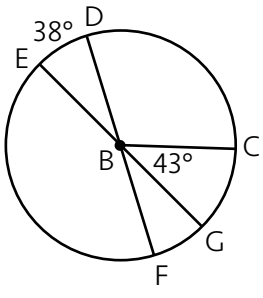
a.



$m\overset{\frown}{BC} =$ _____

$m\overset{\frown}{DEB} =$ _____

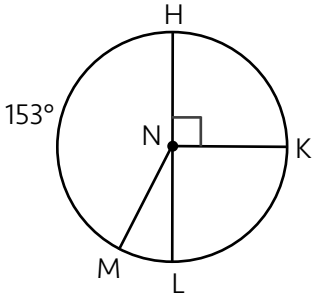
b.



$m\overset{\frown}{DC} =$ _____

$m\overset{\frown}{DFG} =$ _____

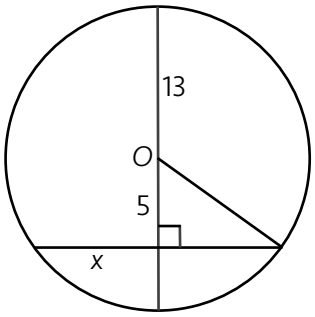
c.



$m\angle MNL =$ _____

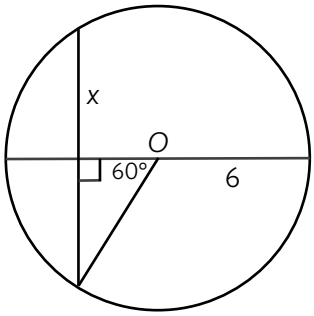
$m\overset{\frown}{KLM} =$ _____

51. Find the value of x in circle O.

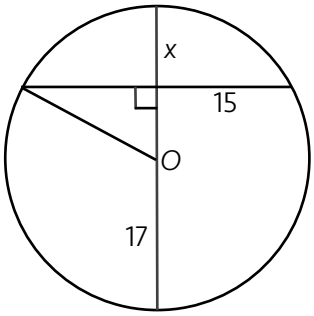


52. Use the previous scenario to find the value of x in each circle, centered at point O.

a.

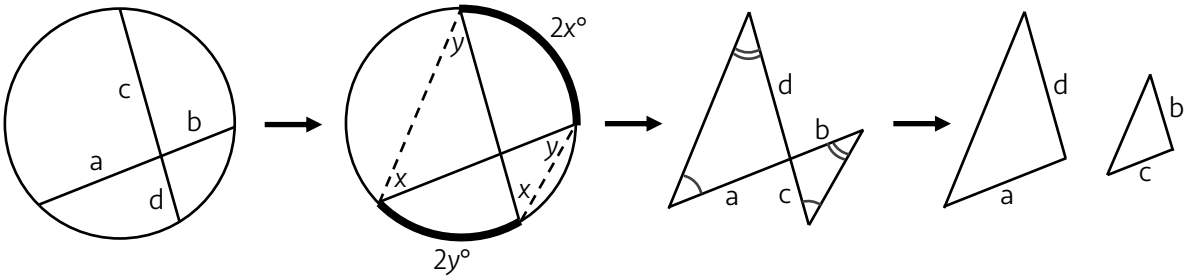


b.



Intersecting Chords, Secants & Tangents

53. When 2 chords intersect <u>inside</u> a circle, the chords are split into proportional segments. To see this relationship, connect the chord endpoints to form two triangles. Using inscribed angle relationships, you can show that the triangles have congruent angles, making them similar.

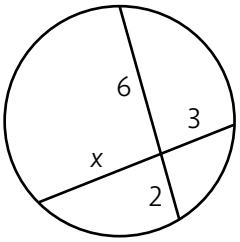


Since the triangles are similar, it follows that $\frac{a}{c} =$

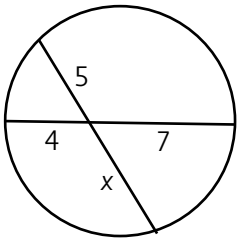
After cross-multiplying, the proportion can be written as $ab =$

54. Use this relationship, ab = cd, to find the value of *x* in each figure.

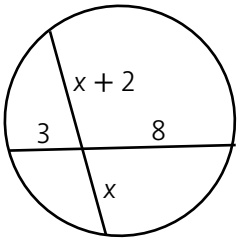
a.



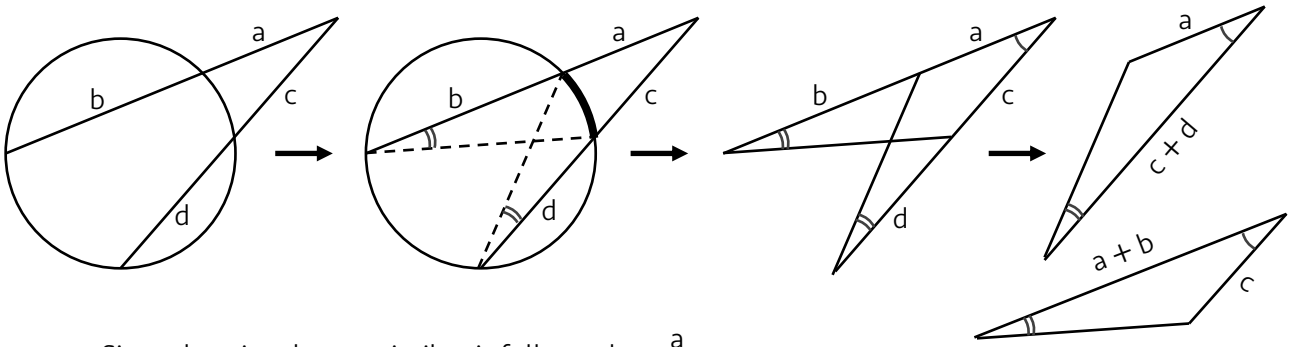
b.



c.



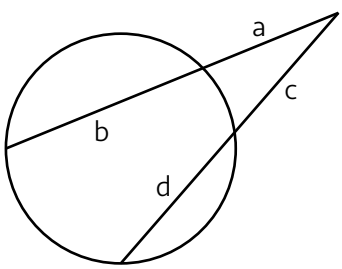
55. When 2 chords intersect <u>outside</u> a circle, they form proportional segments. To see this relationship, form two triangles as shown below. Using inscribed angle relationships, you can show that the triangles have congruent angles, making them similar. Note: if 2 triangles have 2 pairs of congruent angles, their 3rd angles are also congruent.



Since the triangles are similar, it follows that $\frac{a}{c} =$

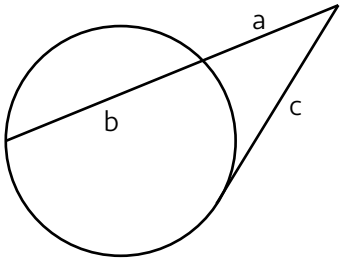
After cross-multiplying, the proportion can be written as $a(a + b) =$

There are 3 variations involving secant and/or tangent lines intersecting outside a circle.



secant–secant

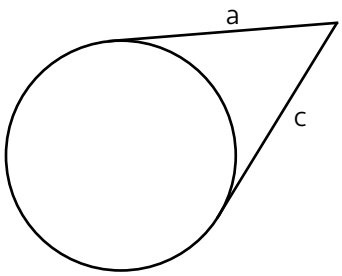
$a(a + b) = c(c + d)$



secant–tangent
$d = 0$

$a(a + b) = c(c + 0)$

$a(a + b) = c^2$



tangent–tangent
$b = d = 0$

$a(a + 0) = c(c + 0)$

$a^2 = c^2 \rightarrow a = c$

34.	a. $95 = \pi r^2 \rightarrow \frac{95}{\pi} = r^2$ $\rightarrow r = \sqrt{\frac{95}{\pi}} = 5.5$ in. b. $141 = \pi r^2 \rightarrow \frac{141}{\pi} = r^2 \rightarrow r = 6.7$ cm
35.	a. $\frac{1}{4} \cdot 24 \rightarrow 6$ cm^2 b. $\frac{1}{8} \cdot 24 \rightarrow 3$ cm^2
36.	a. $\frac{1}{6} \cdot \pi(3)^2 \rightarrow 1.5\pi$ ft^2 b. $\frac{1}{2} \cdot \pi(6)^2 \rightarrow 18\pi$ mm^2 c. $\frac{3}{4} \cdot \pi(7)^2 \rightarrow 36.75\pi$ in.^{2}
37.	$\frac{40°}{360°} \cdot \pi(3)^2 \rightarrow \frac{1}{9} \cdot 9\pi \rightarrow \pi$ in.^{2}
38.	$\frac{130°}{360°} \cdot \pi(19) \rightarrow \approx 21.6$cm
39.	$15\pi = \frac{x}{360°} \cdot \pi(5)^2 \rightarrow \frac{3}{5} = \frac{x}{360°}$ $\rightarrow x = 216°$ The central angle is 216°.
40.	a. Calculate the sector area. Subtract the right triangle area. $\frac{90°}{360°} \cdot \pi(6)^2 - \frac{6 \cdot 6}{2} \rightarrow 9\pi - 18$ cm^2 b. $\frac{60°}{360°} \cdot \pi(4)^2 - \frac{4 \cdot 2\sqrt{3}}{2} \rightarrow \frac{8}{3}\pi - 4\sqrt{3}$ ft^2
41.	$\frac{1}{2} \cdot \pi(7)^2 - \frac{7 \cdot 7\sqrt{3}}{2} \rightarrow 24.5\pi - 24.5\sqrt{3}$ m^2
42.	a. average b. 80° c. 83° d. $\frac{110 + m\widehat{B}}{2} = 78 \rightarrow 110 + m\widehat{B} = 156$ $\rightarrow m\widehat{B} = 46°$
43.	a. $x = \frac{135 + 45}{2} \rightarrow x = 90°$ b. $\frac{88 + 62}{2} \rightarrow 75 \rightarrow x = 180 - 75 = 105°$
44.	a. $m\widehat{NM} = 180° - 103° = 77°$ $\frac{x + 77}{2} = 101 \rightarrow x + 77 = 202 \rightarrow x = 125°$ b. $\frac{88 + x}{2} = 90 \rightarrow 88 + x = 180 \rightarrow x = 92°$
45.	a. $x = \frac{90 - 40}{2} = \frac{50}{2} = 25°$ b. $x = \frac{94 - 52}{2} = \frac{42}{2} = 21°$ c. $34 = \frac{100 - B}{2} \rightarrow 68 = 100 - B$ $\rightarrow B = 32°$
46.	a. $x = \frac{92 - 30}{2} = \frac{62}{2} = 31°$ b. $38 = \frac{x - 44}{2} \rightarrow 76 = x - 44 \rightarrow x = 120$
47.	a. $\frac{2}{3}$; perpendicular lines have opposite reciprocal slopes b. $-\frac{1}{0} \rightarrow$ undefined c. $\overline{ED} \perp \overline{AD} \rightarrow \triangle EDA$ is a right triangle
	$3^2 + 4^2 = (EA)^2 \rightarrow EA = 5$ d. 12
48.	a. radius is $\perp$ to the tangent: $\overline{CD} \perp \overline{DE}$ $\triangle CDE$ is a right triangle; $CD = CF = 8$ $8^2 + x^2 = 17^2 \rightarrow x = \sqrt{225} \rightarrow 15$ b. Since the radius is part of the diameter, the diameter is $\perp$ to the tangent: $\overline{AB} \perp \overline{AC}$ $\triangle CAB$ is a right triangle; hypotenuse = 9 $5^2 + x^2 = 9^2 \rightarrow x = \sqrt{56} \rightarrow 2\sqrt{14}$
49.	$\overline{ED} \perp \overline{DC}$; there are 3 similar right triangles $\frac{FC}{DF} = \frac{DF}{EF} \rightarrow \frac{4}{DF} = \frac{DF}{12} \rightarrow (DF)^2 = 48$ DF: $4\sqrt{3}$ $12^2 + \left(4\sqrt{3}\right)^2 = (DE)^2 \rightarrow$ DE: $8\sqrt{3}$ Since $\triangle DEF$ has sides in a ratio $x : x\sqrt{3} : 2x$, it is a 30°-60°-90° $\triangle$ and $m\angle DEF = 30°$.
50.	a. $\overline{CD}$ or $\overline{CE}$ or $\overline{ED}$ b. isosceles c. right
51.	a. $x = 12$; The right triangle hypotenuse is the circle's radius, 13.
52.	a. $x = 3\sqrt{3}$; The circle's radius, 6, is the hypotenuse of the 30°–60°–90° triangle. b. $x = 9$; The circle's radius is the right triangle's hypotenuse, 17. Its longer leg is 15 and its shorter leg is 8. $x = 17 - 8$.
53.	$\frac{a}{c} = \frac{d}{b} \rightarrow ab = cd$
54.	a. $3 \cdot x = 6 \cdot 2 \rightarrow 3x = 12 \rightarrow x = 4$ b. $5x = 4 \cdot 7 \rightarrow 5x = 28 \rightarrow x = 5.6$ c. $x(x + 2) = 3 \cdot 8 \rightarrow x^2 + 2x = 24$ $\rightarrow x^2 + 2x - 24 = 0$ $\rightarrow (x + 6)(x - 4) = 0 \rightarrow x = -6, 4$ A segment length must be positive so the only valid solution is $x = 4$.
55.	$\frac{a}{c} = \frac{c+d}{a+b} \rightarrow a(a + b) = c(c + d)$
56.	a. $3(3 + 5) = 4(4 + x)$ $\rightarrow 24 = 16 + 4x \rightarrow x = 2$ b. $x(x + 7) = 6(6 + 4)$ $\rightarrow x^2 + 7x = 60 \rightarrow x^2 + 7x - 60 = 0$ $\rightarrow (x + 12)(x - 5) = 0 \rightarrow x = -12, 5$ segment length must be positive $\rightarrow x = 5$ c. $6(6 + x) = 10^2$ $\rightarrow 36 + 6x = 100 \rightarrow x = 10\frac{2}{3}$
57.	a. $4(4 + 5) = x(x + 0)$ $\rightarrow 36 = x^2 \rightarrow x = \pm 6 \rightarrow x = 6$ b. $7(7 + 5) = 6(6 + y)$ $84 = 36 + 6y \rightarrow y = 8$ c. $2x - 3 = x + 7 \rightarrow x = 10$
58.	$x = 6$ $y = 9$ $z = 7$
59.	3
60.	a. 5 b. distance: 5; solve $4^2 + 3^2 = c^2$ c. distance: 5; solve $3^2 + 4^2 = c^2$

Unit 7 Summary

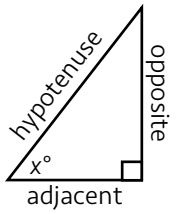
The key terms and concepts in this Unit:

- **Trigonometric Ratios (SOHCAHTOA):**

  Sine: SOH $\sin(x°) = \frac{\text{opp}}{\text{hyp}}$

  Cosine: CAH $\cos(x°) = \frac{\text{adj}}{\text{hyp}}$

  Tangent: TOA $\tan(x°) = \frac{\text{opp}}{\text{adj}}$



  $\text{tangent} = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{rise}}{\text{run}} \rightarrow$ **tangent = slope**

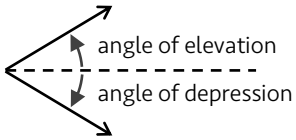
- **Inverse Trigonometry:** Used to find missing angle measures when side lengths are known.

  inverse sine → $\sin^{-1}$ inverse cosine → $\cos^{-1}$ inverse tangent → $\tan^{-1}$

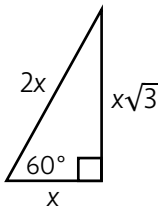
  "the sine of an angle is a ratio" → sin(angle) = ratio

  "the inverse sine of a ratio is an angle" → $\sin^{-1}$(ratio) = angle

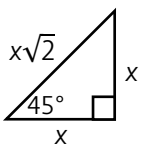
- **Applications:** Solving real-world scenarios involving right triangle relationships, including angles of elevation and depression.
  - **Angle of elevation** is measured upward from horizontal.
  - **Angle of depression** is measured downward from horizontal.



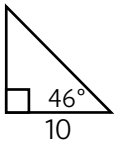
- **Special Right Triangle Ratios:**
  - **30-60-90 Trig Ratios**
    - $\sin30° = \cos60° = \frac{1}{2}$
    - $\cos30° = \sin60° = \frac{\sqrt{3}}{2}$
    - $\tan30° = \frac{\sqrt{3}}{3}$
    - $\tan60° = \sqrt{3}$



  - **45-45-90 Trig Ratios**
    - $\sin45° = \cos45° = \frac{\sqrt{2}}{2}$
    - $\tan45° = 1$

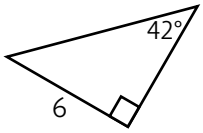


14. The previous triangle is *almost* a 30-60-90 triangle, so its hypotenuse length must be almost 10 units. Consider a different triangle, with a 46° angle. Estimate the length of its hypotenuse. Round to the nearest integer.

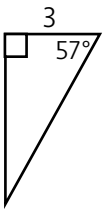


15. Estimate the length of the hypotenuse in each triangle shown.

a.



b.

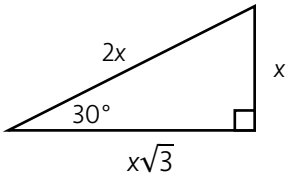


Using what you currently know, you can only identify the missing lengths in the previous scenario by estimating. Since the angles in the previous triangles are close to ones you have learned about, you can estimate accurately. If the angles are not close to 30, 45, or 60, estimating is difficult.

A better method for finding sides and angles of right triangles uses a calculator and ratios. You know the ratios of special right triangles, but you don't know the ratios for every right triangle. For other triangles, you can use a calculator.

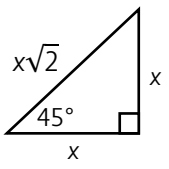
The Tangent Ratio

16. You can use **ratios** to calculate the side lengths in special right triangles.
In every 30-60-90 triangle, the sides have a ratio of $x : x\sqrt{3} : 2x$.
Write this ratio as 3 separate ratios below.



a. $\frac{\text{shorter leg}}{\text{hypotenuse}}$ b. $\frac{\text{longer leg}}{\text{hypotenuse}}$ c. $\frac{\text{shorter leg}}{\text{longer leg}}$

17. In every 30-60-90 triangle, the sides have a ratio of $x : x : x\sqrt{2}$.
Write this ratio as 2 separate ratios below.



a. $\frac{\text{leg}}{\text{hypotenuse}}$ b. $\frac{\text{vertical leg}}{\text{horizontal leg}}$

18. In a 45-45-90 triangle, the ratio of the vertical leg to the horizontal leg is 1:1. To simplify the wording, this ratio is called the **tangent** ratio. Read the three questions below to see how mathematicians use the word tangent.

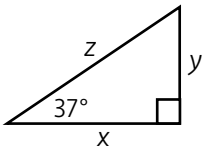
> Original: In a 45-45-90 triangle, what is the ratio of the vertical leg to the horizontal leg?
>
> Concise: In a 45-45-90 triangle, what is the tangent of 45°?
>
> More concise: What is the tangent of 45°?

After reading the previous questions, write the phrase that the word tangent replaces.

19. A ratio can be written as a fraction. In the space below, write the tangent ratio as a fraction.

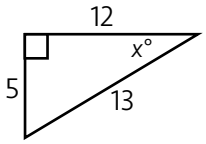
tangent = _____

20. What is the tangent of 37° in the triangle shown?

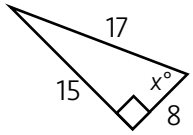


21. The previous right triangle is shown in a standard position, with one leg vertical and one leg horizontal. When a right triangle is not arranged this way, it is harder to find the tangent ratio. Try to identify the tangent of $x°$ in each right triangle below.

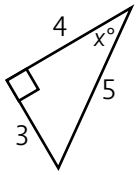
a.



b.



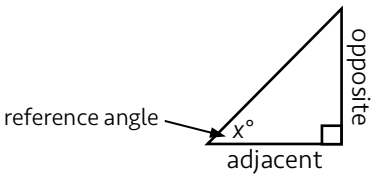
c.



22. Since triangles are not always in a standard position with a vertical side, the words vertical and horizontal are replaced with new words: opposite and adjacent ("uh-JAY-sent"). The **opposite side** in a right triangle is directly across from the reference angle. The tangent ratio is defined below. Write the tangent ratio 2 more times.

$$\text{tangent of } x° = \frac{\text{opposite side}}{\text{adjacent side}}$$

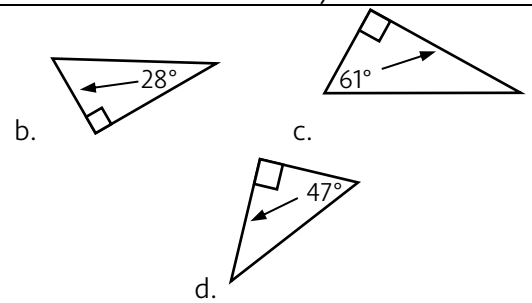
tangent of $x°$ = _____



tangent of $x°$ =

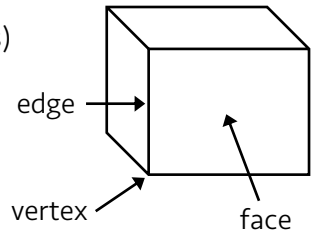
Answer Key

1.	a. $3^2 + 4^2 = c^2 \rightarrow c^2 = 25 \rightarrow c = 5$ b. $2^2 + b^2 = 4^2 \rightarrow b^2 = 12 \rightarrow b = 2\sqrt{3}$ c. $2^2 + 2^2 = c^2 \rightarrow c^2 = 8 \rightarrow c = 2\sqrt{2}$
2.	Not enough information
3.	$x = 45$
4.	a. leg: x hypotenuse: $x\sqrt{2}$ b. both legs: x
5.	a. leg: 4; hypotenuse: $4\sqrt{2}$ b. leg: 7; hypotenuse: $7\sqrt{2}$ c. both legs: 11
6.	20 inches; each side is 5 inches long
7.	a. hypotenuse: $1\sqrt{2}$ or $\sqrt{2}$ b. both legs: $\frac{8}{\sqrt{2}} \rightarrow \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{8\sqrt{2}}{2} \rightarrow 4\sqrt{2}$ c. both legs: $\frac{1}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{1\sqrt{2}}{2} \rightarrow \frac{\sqrt{2}}{2}$
8.	$x = 60$ $y = 30$
9.	$2\sqrt{3}$ cm; Draw the height as a dashed line, which forms a right triangle. Use the Pythagorean Theorem to find the height. Solve $2^2 + b^2 = 4^2 \rightarrow b^2 = 12 \rightarrow b = 2\sqrt{3}$.
10.	a. shorter leg: x; longer leg: $x\sqrt{3}$ b. shorter leg: x; hypotenuse: $2x$
11.	a. longer leg: $4\sqrt{3}$ b. shorter leg: 10; longer leg: $10\sqrt{3}$ c. shorter leg: 8; hypotenuse: 16
12.	a. shorter leg: $\frac{9}{\sqrt{3}} \rightarrow \frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{9\sqrt{3}}{3} \rightarrow 3\sqrt{3}$ hypotenuse: $2 \cdot 3\sqrt{3} \rightarrow 6\sqrt{3}$ b. shorter leg: $\frac{1}{2}$ longer leg: $\frac{1}{2} \cdot \sqrt{3} \rightarrow \frac{\sqrt{3}}{2}$
13.	About 10 units
14.	This triangle is <u>almost</u> a 45-45-90 triangle. hypotenuse $\approx 10\sqrt{2} \approx 14$ units
15.	a. About $6\sqrt{2}$ units b. About 6 units
16.	a. $\frac{x}{2x} \rightarrow \frac{1}{2}$ b. $\frac{x\sqrt{3}}{2x} \rightarrow \frac{\sqrt{3}}{2}$ c. $\frac{x}{x\sqrt{3}} \rightarrow \frac{1}{\sqrt{3}} \rightarrow \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{\sqrt{3}}{3}$
17.	a. $\frac{x}{x\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$ b. $\frac{x}{x} \rightarrow \frac{1}{1}$
18.	"tangent" replaces the phrase "the ratio of the vertical leg to the horizontal leg"
19.	tangent $= \frac{\text{vertical leg}}{\text{horizontal leg}}$
20.	tangent of $37° = \frac{y}{x}$

21.	a. $\frac{5}{12}$ b. $\frac{15}{8}$ c. $\frac{3}{4}$
22.	tangent of $x° = \frac{\text{opposite side}}{\text{adjacent side}}$
23.	
24.	a. side b b. side e c. side h
25.	The tangent ratio is $\frac{\text{opposite side}}{\text{adjacent side}}$.
26.	a. side y b. side x c. side y
27.	a. $\frac{b}{a}$ b. $\frac{e}{d}$ c. $\frac{h}{g}$
28.	a. $\tan(x°) = \frac{3}{4}$ b. $\tan(x°) = 2$
29.	a. $\frac{b}{c}$ b. $\frac{f}{d}$ c. $\frac{g}{h}$
30.	a. $y°$ b. $y°$ c. $x°$
31.	a. $\tan 34° = \frac{w}{10}$ b. $\tan 50° = \frac{17}{x}$ c. $\tan 28° = \frac{y}{6}$
32.	There are 6 possible ratios. $\frac{O}{A}$ $\frac{A}{O}$ $\frac{O}{H}$ $\frac{H}{O}$ $\frac{A}{H}$ $\frac{H}{A}$
33.	a. sine of $x° = \frac{\text{opposite}}{\text{hypotenuse}}$ b. cosine of $x° = \frac{\text{adjacent}}{\text{hypotenuse}}$
34.	a. $\sin(x°) = \frac{\text{opp}}{\text{hyp}}$ b. $\cos(x°) = \frac{\text{adj}}{\text{hyp}}$ c. $\tan(x°) = \frac{\text{opp}}{\text{adj}}$
35.	a. $\sin(x°) = \frac{\text{opp}}{\text{hyp}}$ b. $\cos(x°) = \frac{\text{adj}}{\text{hyp}}$ c. $\tan(x°) = \frac{\text{opp}}{\text{adj}}$
36.	a. S → sine b. C → cosine c. T → tangent
37.	a. O → opposite b. H → hypotenuse c. A → adjacent
38.	a. $\frac{O}{H}$ b. $\frac{A}{H}$ c. $\frac{O}{A}$
39.	$\sin(x°) = \frac{O}{H}$ $\cos(x°) = \frac{A}{H}$ $\tan(x°) = \frac{O}{A}$
40.	a. $\sin(x°) = \frac{3}{4}$ b. $\cos(x°) = -\frac{1}{5}$

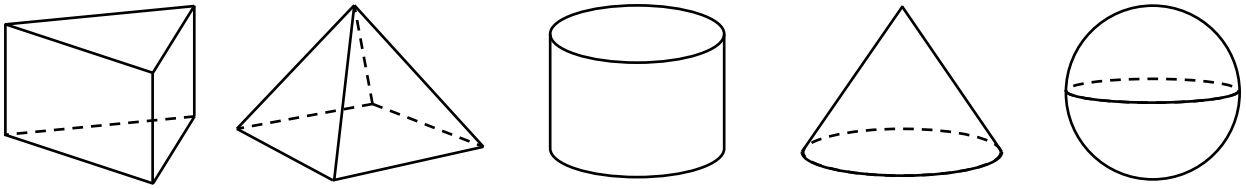
Unit 8 Summary

The key terms and concepts in this Unit:



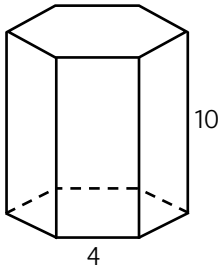
- **3D Fundamentals:** Solids are defined by faces (surfaces), edges (segments) and vertices (points).
  - **Euler's Formula:** $F + V = E + 2$.
- **Prism:** A solid shape with two parallel, congruent faces called bases.
- **Pyramid:** A polygon base and triangular faces that meet at a point.
- **Cylinder:** A prism with circular bases.
- **Cone:** A circular base that tapers to a point at the top.
- **Sphere:** The set of all points equidistant from a center point.

Prism | Pyramid | Cylinder | Cone | Sphere



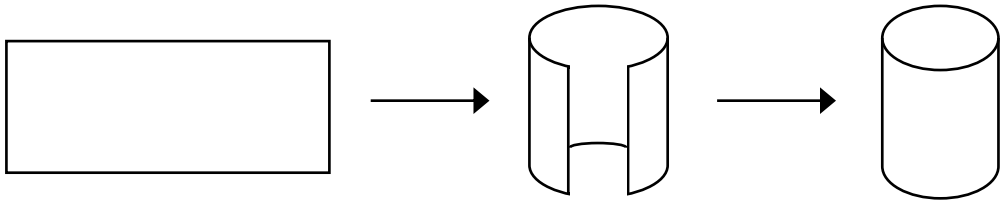
- **Surface Area (SA):** The total area of all faces.
  - **Prisms:** Add the areas of all the faces. → 2 parallel n-sided bases + n rectangular sides
  - **Cylinders:** $SA = 2\pi r^2 + 2\pi rh$ → 2 circles + 1 rectangle
  - **Pyramid:** Add the areas of all the faces. → 1 n-sided base + n isosceles triangles
  - **Cone:** $SA = \pi r^2 + \pi rL$
  - **Sphere:** $SA = 4\pi r^2$
- **Lateral Surface Area (LSA):** The area of the sides, without the bases.
- **Volume (V):** The cubic space inside a solid.
  - **Prisms:** $V = Bh$ → Base Area × height
  - **Cylinders:** $V=Bh$ → $V = \pi r^2 h$
  - **Pyramid:** $V = \frac{1}{3}Bh$
  - **Cone:** $V = \frac{1}{3}Bh$ → $V = \frac{1}{3}\pi r^2 h$
  - **Sphere:** $V = \frac{4}{3}\pi r^3$
- **Volume Ratios of Similar Figures**
  - If the scale factor is $x : y$, then the volume ratio is $x^3 : y^3$.

10. What is the surface area of the hexagonal prism? Its base is a regular hexagon.



Surface Area of a Cylinder

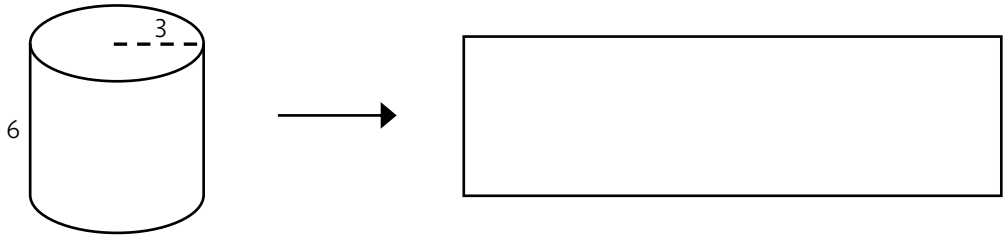
11. A prism with circular bases is called a **cylinder**. To make a cylinder, start with a rectangular sheet of paper. Choose opposite edges and curl them together until the edges meet, making the 2D rectangle become a 3D cylinder. The length of the rectangle's base becomes the circle's circumference.



a. If a rectangle has a height of 10 cm, the height of the cylinder it forms is _____.

b. If a rectangle forms a cylinder with a radius of 4 cm, how wide was the rectangle's base?

12. What are the dimensions of the rectangle formed by opening up the cylinder shown?

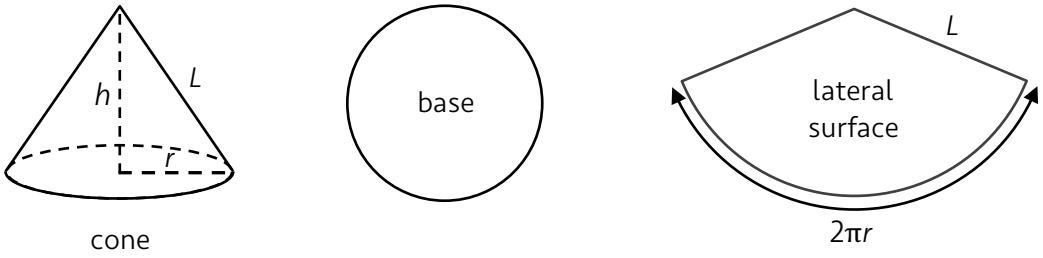


base: _____ height: _____

13. The previous rectangle is called the lateral part of the cylinder. The cylinder's **lateral surface area** is the area of its sides, without the bases. What is the lateral surface area of a cylinder with a height of 10 inches and a base radius of 7 inches.

Surface Area of a Cone

24. A **cone** has a circular base and a point at the top. The base has a radius, r, and a height, h. A cone also has a **slant height**, L, which is the distance from the base's edge to the top of the cone.



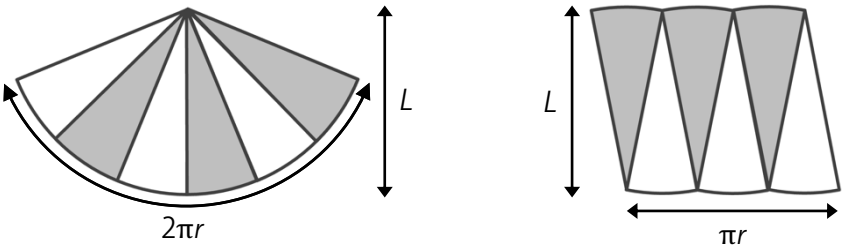
cone

a. The area of the base is _____.

b. The lateral surface of the cone, when flattened, looks like the shape above. Why does the curved portion of the lateral surface have a length of $2\pi r$?

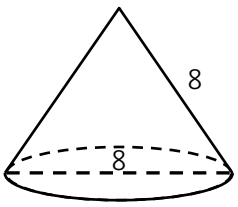
c. The lateral surface is shown below, split into 6 sections and rearranged to form a parallelogram. The parallelogram's base is half of the circumference, πr. Its height is L, the cone's slant height.

The parallelogram's area is _____.

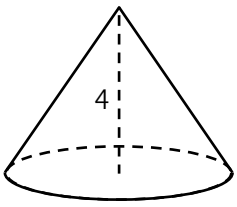


25. The surface area of a cone is $SA = \pi r^2 + \pi rL$. Use this formula to find the surface area of each cone.

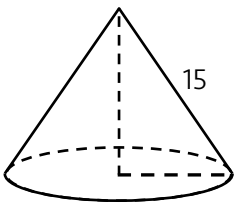
a. diameter = 8 in.



b. radius = 3 cm



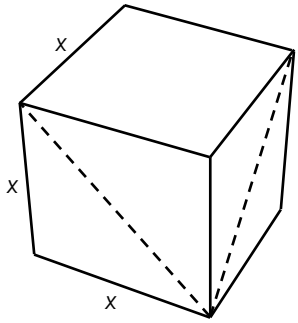
c. height = 12 m



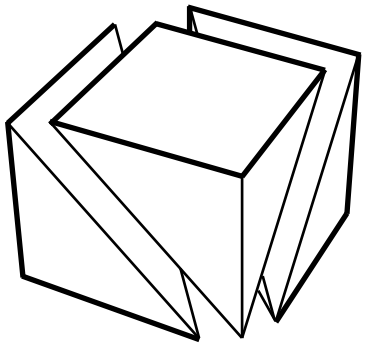
46. The formula for a cylinder's volume is _____.

Volume of a Pyramid

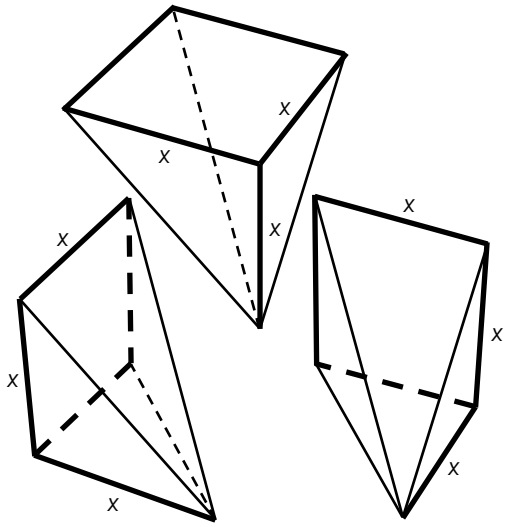
47. The volume of a **pyramid** is related to the volume of a prism. One way to see this relationship is to divide a cube into three congruent pyramids, as shown.



One Cube



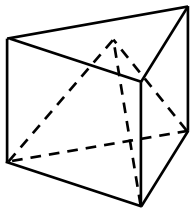
Three Pyramids



a. What is the volume of the cube?

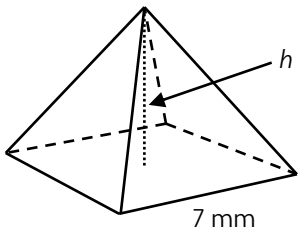
b. What is the volume of each pyramid?

48. A pyramid is $\frac{1}{3}$ of a prism's volume, if they have congruent bases and heights.



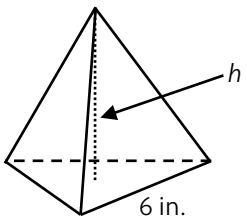
Since a prism's volume is $V = Bh$, a pyramid's volume is $V = \frac{1}{3}Bh$, if B is the base's area and h is its height. The previous example does not prove the volume formula for <u>all</u> pyramids, but it can help you remember it. Use this formula to find each pyramid's volume, if its base is a regular polygon.

a.



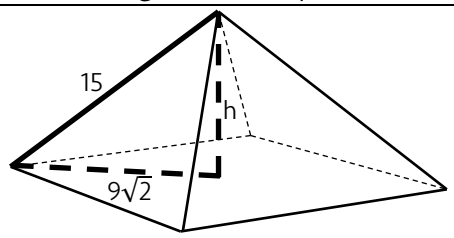
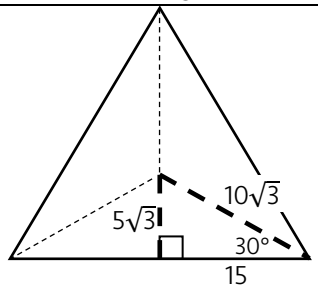
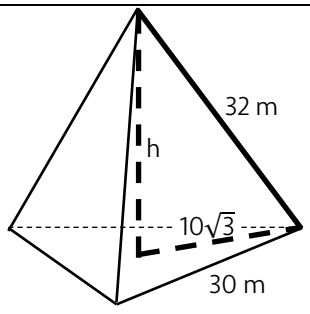
height, h: 6 mm

b.



height: 8.4 in.

28.	$SA = \pi r^2 + \pi rL$
29.	a. $SA = 4\pi(3)^2 = 36\pi$ cm^2 b. $4\pi(5)^2 = 100\pi$ in.^{2}
30.	$4\pi(7.5)^2 = 706.9$ ft^2
31.	a. $144\pi = 4\pi r^2 \rightarrow r = 6$ cm b. $32 = 4\pi r^2 \rightarrow r = \sqrt{\frac{8}{\pi}} \approx 1.6$ in.
32.	$C = 2\pi r \rightarrow 43.98 = 2\pi r \rightarrow r = 7$ $SA = 4\pi(7)^2 = 196\pi$ cm^2
33.	$SA = 4\pi r^2$
34.	3 rows of 4, or 4 rows of 3 $\rightarrow$ 12 cubes
35.	a. 4 layers of 18 $\rightarrow$ 72 cubes b. $3 \cdot 6 \cdot 4 \rightarrow$ 72 cubic inches
36.	a. $7 \cdot 4 \rightarrow$ 28 square feet b. $7 \cdot 6 \rightarrow 42$ m^2
37.	a. $28 \cdot 5 \rightarrow$ 140 cubic feet b. $42 \cdot 20 \rightarrow 840$ m^3
38.	a. The base area is $\frac{1}{2}$bh $\rightarrow \frac{1}{2} \cdot 6 \cdot 8 \rightarrow 24$ The volume is 24 mm$^2 \cdot 11$ mm $\rightarrow 264$ mm^3 b. An equilateral triangle with a base of 8 has a height of $4\sqrt{3}$. Its area is $16\sqrt{3}$. The volume is $16\sqrt{3}$ yd$^2 \cdot 7$ yd $\rightarrow 112\sqrt{3}$ yd^3
39.	The base is a hexagon, which can be split into 6 equilateral triangles. Each triangle's base is 10, height is $5\sqrt{3}$, and area is $25\sqrt{3}$. The hexagon's area is $25\sqrt{3} \cdot 6 \rightarrow 150\sqrt{3}$. The hexagonal prism's volume is $150\sqrt{3}$ in.$^2 \cdot 22$ in. $\rightarrow 3{,}300\sqrt{3}$ in.^{3}
40.	10 mm$^3 \cdot 5$ discs $\rightarrow 50$ mm^3
41.	a. πr^2 b. $V = \pi r^2 \cdot h \rightarrow \pi r^2 h$
42.	a. base: 18 cm^2 height: 4 cm volume: 18 cm$^2 \cdot 4$ cm $\rightarrow 72$ cm^3 b. base area: 16π m^2 height: 1 m volume: 16π m$^2 \cdot 1$ m $\rightarrow 16\pi$ m^3 c. base area: 25π in.^{2} height: 8 in. volume: 25π in.$^2 \cdot 8$ in. $\rightarrow 200\pi$ in.^{3}
43.	a. $V = \pi(4)^2 \cdot (7) \rightarrow 112\pi$ in.^{3} b. $V = \pi(3)^2 \cdot (4) \rightarrow 36\pi$ cm^3 c. $V = \pi(4.5)^2 \cdot (12) \rightarrow 243\pi$ m^3
44.	$931.5\pi = \pi(9)^2 h \rightarrow 931.5\pi = 81\pi h$ $\rightarrow h = 11.5$ yards
45.	$450.1 = \pi r^2(7.4) \rightarrow 19.361 = r^2$ $\rightarrow r = 4.4$ feet
46.	$V = \pi r^2 h$
47.	a. x^3 b. $\frac{1}{3}x^3$

48.	a. The base area, B, is $7^2 \rightarrow 49$. $V = \frac{1}{3}(49)(6) \rightarrow V = 98$ mm^3 b. The base is an equilateral triangle. Its area, B, is $\frac{1}{2}(6)(3\sqrt{3}) \rightarrow 9\sqrt{3}$. $V = \frac{1}{3}(9\sqrt{3})(8.4) \rightarrow V = 25.2\sqrt{3}$ in.^{3}
49.	a. shorter leg: 6 $\rightarrow$ half of square base edge longer leg: 8 $\rightarrow$ solve: $6^2 + b^2 = 10^2$ b. both legs: $6\sqrt{2} \rightarrow$ it is a 45-45-90 triangle
50.	a. base: $6\sqrt{2} \rightarrow$ see previous scenario height: $2\sqrt{7} \rightarrow$ solve: $\left(6\sqrt{2}\right)^2 + b^2 = 10^2$ b. base: 6 $\rightarrow$ half of square base's edge hypotenuse: 8 $\rightarrow$ see previous scenario height: $2\sqrt{7} \rightarrow$ solve: $6^2 + b^2 = 8^2$ You can also find the height using the right triangle shown in part a.
51.	 Using a 45-45-90 triangle, the base of the dashed right triangle shown is $9\sqrt{2}$. height: $3\sqrt{7} \rightarrow$ solve: $\left(9\sqrt{2}\right)^2 + h^2 = 15^2$ Area of base, B: $18^2 \rightarrow 324$ ft^2 Volume: $\frac{1}{3} \cdot 324 \cdot 3\sqrt{7} \rightarrow 324\sqrt{7}$ ft^3
52.	 The pyramid's base is an equilateral $\triangle$. The $10\sqrt{3}$ side base of the right $\triangle$ highlighted below.  pyramid height: $\sqrt{32^2 - \left(10\sqrt{3}\right)^2} \rightarrow \sqrt{724}$ Pyramid's base area, B: $\frac{1}{2}(30)(15\sqrt{3}) \rightarrow 225\sqrt{3}$ m^2 Volume: $\frac{1}{3}(225\sqrt{3})(\sqrt{724}) \approx 3{,}495.4$ m^3

UNIT TESTS

The Path to Mastery

Each Unit in this book guides you along a path toward mastery, building your knowledge of Geometry as you analyze patterns, develop new skills and discover important relationships and formulas.

Use these 8 Unit Tests to assess your progress and identify areas for growth. As you work through each test, think about the ideas you have learned and how to apply them. Draw a diagram to help you visualize. Estimate a realistic result before solving. Look for parallel lines, congruent parts, or proportional sides. Remember, getting lost and making mistakes can help you find a better path.

Tips for Success

Follow these guidelines to accurately measure your progress:

1. **Work Independently:** Try to complete each test without looking back in the book.
2. **Show Every Step:** The path to the solution is important. If you put your thoughts down on the page by writing out your steps, you may find a clearer path to the solution.
3. **Check Yourself:** Always ask: "Does this make sense?" If you solve for the side of a triangle and the length is negative, or if an angle is obtuse but has a measure of 20°, these are clues that can help you spot mistakes in your logic.
4. **Learn From Mistakes:** After using the Answer Key to score your test, go back through that Unit and review concepts you missed to help you understand them better.

Mastery Chart

After each Unit Test, mark the scenarios you understand well in the chart below. Go back through the Unit to review the concepts you are still learning or come back to them later if you are ready to start the next Unit. Adjust this chart as you progress through the book and your mastery grows.

	1	2	3	4	5	6	7	8	9	10
Unit 1										
Unit 2										
Unit 3										
Unit 4										
Unit 5										
Unit 6										
Unit 7										
Unit 8										

Unit 1 Test

1. Points A(−6, 8), B(−1, k) and C(9, −1) are collinear. Find the value of k.

2. $\overline{PT}$ has a midpoint at (5, −7). If P is located at (−2, 3), find the coordinates of T.

3. The endpoints of a circle's diameter are (3, −6) and (−5, −2).

   a. Where is the circle's center point?

   b. How long is its radius?

4. Point M is between L and N on a segment. If LM = $x^2 + 2$, MN = 6, and LN = $x + 28$, solve for x and find the length of $\overline{LM}$.

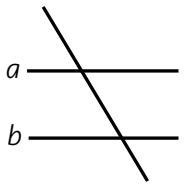
5. Angles 1 and 2 are complementary. If $m\angle 2$ is three times larger than $m\angle 1$, find the measure of $\angle 2$.

6. $\angle 1$ and $\angle 2$ form a linear pair. Find the measure of each angle if $m\angle 1 = (7x - 2)°$ and $m\angle 2 = (3x - 8)°$.

7. $\overline{FB}$ bisects $\angle ABC$. $m\angle ABF = (2x + 5)°$ and $m\angle ABC = (6x - 20)°$. What $m\angle ABF$?

8. $\angle A$ and $\angle B$ are alternate interior angles between 2 parallel lines. If $m\angle A = (4x + 20)°$ and the supplement of $\angle B$ measures 60°, what is the value of x?

9. In the figure, $a \parallel b$. Two corresponding angles measure $(82 - 2x)°$ and $(72 - 3y)°$. Two same-side interior angles measure $(3x + 90)°$ and $(y + 64)°$. Find the measure of the obtuse angles in the figure.



10. A segment has endpoints at A(4, −2) and B(−6, −7).

    a. What is the length of $\overline{AB}$?

    b. What is the equation of the segment's perpendicular bisector?

Unit 1 Test Answer Key

1.	Make the slope between points 1 and 2 equal the slope between points 2 and 3. Solve: $\frac{k-8}{-1+6}=\frac{-1-k}{9+1} \rightarrow \frac{k-8}{5}=\frac{-1-k}{10}$ $\rightarrow 10k-80=-5-5k \rightarrow k=5$
2.	P to midpoint: add 7 to x, subtract 10 from y point T: $(5+7,-7-10) \rightarrow (12,-17)$
3.	a. The center is the midpoint: $(-1,-4)$ b. The radius is the distance between the center point and either endpoint: $d=\sqrt{(3+1)^2+(-6+4)^2}=\sqrt{20}=2\sqrt{5}$
4.	$LM+MN=LN \rightarrow (x^2+2)+6=x+28$ $\rightarrow x^2-x-20=0$ $\rightarrow (x-5)(x+4)=0 \rightarrow x=5$ or -4 if $x=5$, $LM=x^2+2=5^2+2=27$ if $x=-4$, $LM=x^2+2=(-4)^2+2=18$
5.	$m\angle 1+m\angle 2=90 \rightarrow x+3x=90$ $\rightarrow x=22.5 \quad m\angle 2=3(22.5)=67.5°$
6.	$(7x-2)+(3x-8)=180 \rightarrow x=19$ $m\angle 1=7(19)-2=133-2=131°$ $m\angle 2=180-131=49°$
7.	$2(2x+5)=6x-20 \rightarrow x=15$ $m\angle ABF=2(15)+5=35°$
8.	$m\angle ABF=180-60=120°$ $4x+20=120 \rightarrow x=25$
9.	$82-2x=72-3y \rightarrow 2x-3y=10$ $3x+90+y+64=180 \rightarrow 3x+y=26$ Solve the system of equations $2x-3(26-3x)=10 \rightarrow x=8, y=2$
10.	a. $\sqrt{(-6-4)^2+(-7+2)^2}=5\sqrt{5}$ b. midpoint: $(-1,-4.5)$ $\overline{AB}$ slope: $\frac{-5}{10}=-\frac{1}{2}$ $\perp$ bisector equation: $y+4.5=2(x+1)$

Next Steps

After grading the Unit Test, mark the concepts you understand well. If you want to review any concepts on this test, use the guide below to find their location in the book.

Scenario	Concept Location	Notes & Insights
1	p. 8: #16	
2	p. 11: #25	
3	p. 13: #37	
4	p. 12: #30	
5	p. 17: #56	
6	p. 18: #59	
7	p. 22: #78	
8	p. 26: #93	
9	p. 26: #93	
10	10a: p. 12: #36 10b: p. 30: #110	