SUMMIT MATH

Learn at your **OWN** pace.

ADVANCED ALGEBRA & TRIGONOMETRY

SERIES

BOOK 1

UNIT 1: ANALYZING FUNCTIONS & TRANSFORMATIONS

UNIT 2: ABSOLUTE VALUE FUNCTIONS, EQUATIONS & INEQUALITIES

INTRODUCTION

Learning math through Guided Discovery:
Guided Discovery is designed to help you experience a sense of discovery as you learn each topic.

Why this curriculum series is named Summit Math:
Learning through Guided Discovery is like hiking. Learning and hiking both require effort and persistence and people naturally move at different paces, but they can reach the summit if they keep moving forward. Whether you race rapidly through the book or step slowly through each scenario, this textbook is designed to keep advancing your learning until you reach the summit.

Guided Discovery Scenarios:
The Guided Discovery Scenarios in this book are written and arranged to show you that new math concepts are related to previous concepts you already know. Try each scenario on your own first, check your answer when you finish, and then fix any mistakes, if needed. Making mistakes and struggling are essential parts of the learning process.

Homework & Extra Practice Scenarios:
After you complete the scenarios in each Guided Discovery section, extra practice will help you develop better memory of each topic. Use the Homework & Extra Practice Scenarios to improve your understanding and to increase your retention.

The Answer Key:
The Answer Key is included to give you teacher-like guidance. When you finish a scenario, you can get immediate feedback. If the Answer Key does not help you fully understand the scenario, try to get additional guidance from another student or a teacher.

Find more resources at:
www.summitmath.com

SAMPLE PAGES

GUIDED DISCOVERY SCENARIOS

The Guided Discovery Scenarios are designed to help you experience a sense of discovery as you learn. Explanations are brief and carefully timed to allow you to build your learning at a comfortable pace. The Answer Key supports your learning by giving you immediate feedback and helping you understand each step before moving on.

As you complete each scenario, follow these steps.

Step 1: Try the scenario.
Read the scenario carefully and try to work through it. Be willing to struggle.

Step 2: Check the Answer Key.
The Answer Key can guide you through the scenario, show you new ideas, or help you find mistakes in your steps.

Step 3: Fix your mistakes, if needed.
Mistakes are learning opportunities. If your result does not match the Answer Key, check your work and look for errors. If you still need guidance, seek help from a classmate or teacher.

When you are ready, try the next scenario and repeat this cycle.

CONTENTS

Unit 1
Analyzing Functions & Transformations

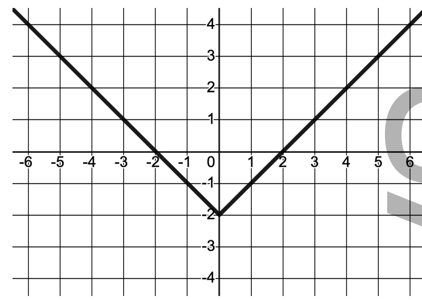
Mini-Unit: Transformations of Functions

Section 1
Functions

Use this page for taking notes or anything else that helps you learn.

Many functions can be modeled by 2-variable equations, but an equation is only called a **function** if each input produces only 1 output.

5. One way to identify a function is to look at its graph. Consider the graph formed by the equation $y = |x| - 2$. Use the graph to fill in the blanks below.



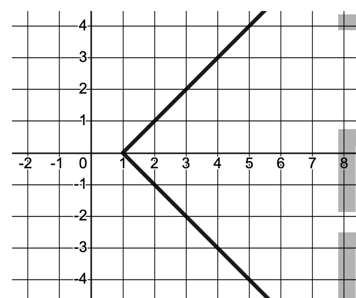
a. If $x = 4$, $y =$____

b. If $x = 0$, $y =$____

c. If $x = -2$, $y =$____

6. Is the previous graph a function? Does each input produce only 1 output? To state this another way, is every point on the graph the only point with that x-value?

7. Now look at the graph formed by the equation $x = |y| + 1$. Use the graph to fill in the blanks below.



a. If $x = 1$, $y =$____

b. If $x = 2$, $y =$____

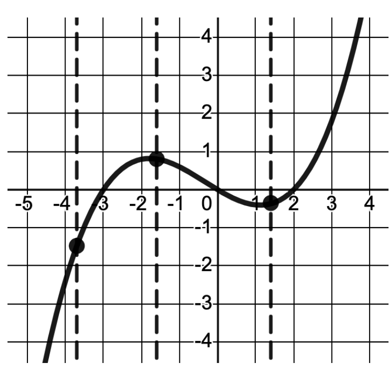
c. If $x = 3$, $y =$____

8. Is the previous graph a function? Does each input produce only 1 output?

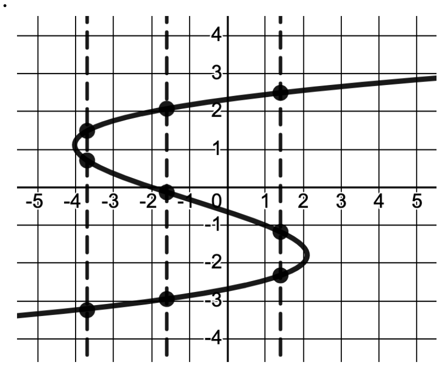
The Vertical Line Test

9. A graph is **not** a function if 2 or more points have the same x-value and different y-values. One way to see this is to imagine a vertical line that moves left and right. If the vertical line ever intersects the graph at 2 or more points, the graph is not a function. It fails the **vertical line test**. Consider each graph shown. Does each graph <u>pass</u> the vertical line test?

a.

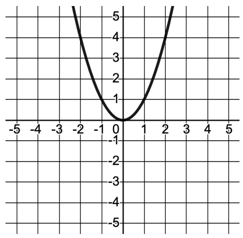


b.

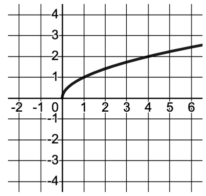


10. Each equation shown has its graph below it. Does each graph pass the vertical line test?

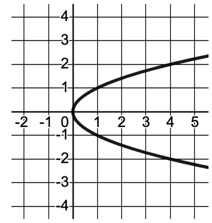
a. $y = x^2$



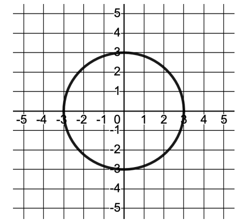
b. $y = \sqrt{x}$



c. $x = y^2$



d. $x^2 + y^2 = 9$



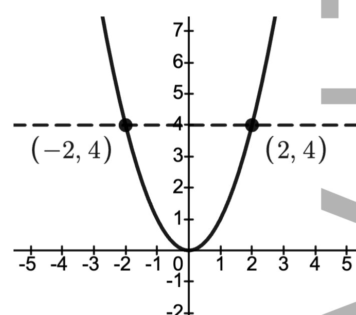
11. Try to sketch a simple graph of each equation below and use the vertical line test to determine if the equation represents a function.

a. $y = -3x$ b. $x = 3$ c. $y = x^2 - 3$ d. $x^2 + y^2 = 4$

1-to-1 Functions & the Horizontal Line Test

A functions is also called **1-to-1** if each output is produced by only 1 input. Consider the function $y = x^2$. If $x = 2$, the output is 4. If $x = -2$, the output is also 4. The output of 4 is produced by more than 1 input so this function is **not** a 1-to-1 function.

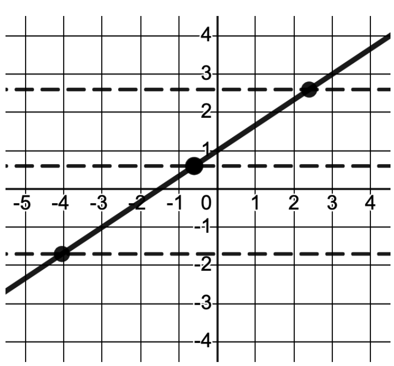
When a function is 1-to-1, every y-value is linked to only 1 x-value. One way to see this is to imagine a horizontal line that moves up or down. If the horizontal line ever intersects the graph at 2 or more points, the function is not 1-to-1. It fails the **horizontal line test**.



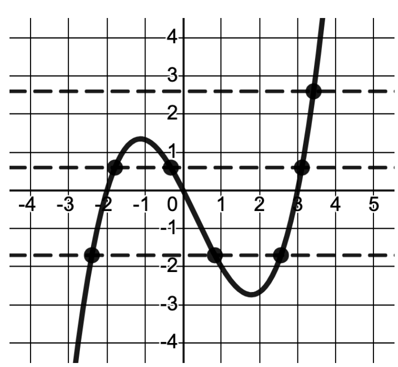
not 1-to-1

12. Consider each graph shown. Does each graph pass the horizontal line test?

a.



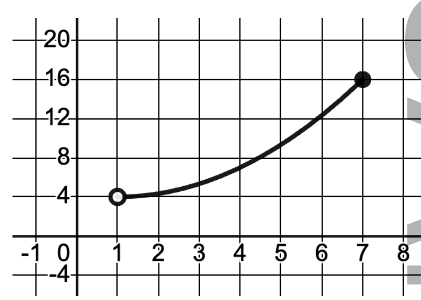
b.



Domain & Range using Interval Notation

Each function has a set of inputs and outputs. The inputs of a function are called the **domain**. The outputs are called the **range**. For some functions, the domain and/or range is "all real numbers." Other functions have restricted domains and/or ranges. It is easier to identify the domain and range when you see the graph of a function.

16. Consider the function in the graph. It contains points with every x-value between 1 and 7, including 7. Using inequality notation, its domain can be expressed as shown below:



domain: $1 < x \leq 7$

What is the function's range?

range:

The inequality $1 < x \leq 7$ can be written more concisely with interval notation as $(1, 7]$. A curved parenthesis "(" shows a number is not included in the interval, while a square bracket "[" means a number is included. When a point is not filled in, the function is not defined at that point.

17. Rewrite each inequality using interval notation.

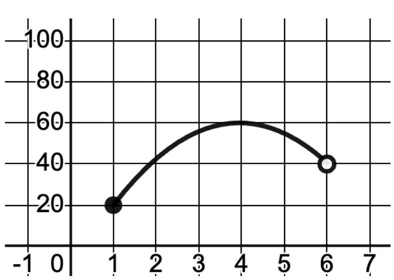
a. $4 \leq x < 10$ b. $-8.2 < x \leq 0$ c. $-7 \leq x < \infty$ d. $x < 2$

18. Rewrite each interval using inequality notation.

a. $[2, 11)$ b. $(-\infty, 3]$ c. $[-10, \infty)$

19. Identify the domain and range of each function shown below.

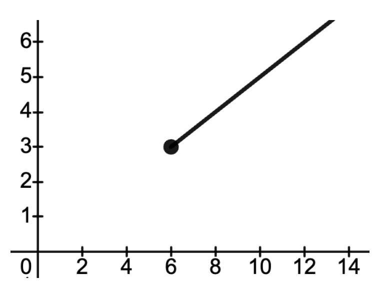
a.



domain:

range:

b.



domain:

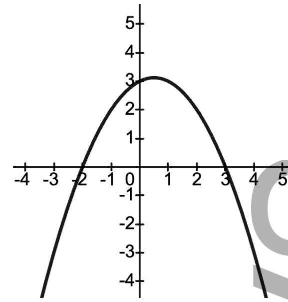
range:

Section 2
Analyzing Functions

Use this page for taking notes or anything else that helps you learn.

Positive & Negative Intervals

1. A function is **positive** when its y-values are positive, when the function is above the x-axis. A function is **negative** when it is below the x-axis. Consider the function shown. It is negative when x is less than -2 or greater than 3. Using interval notation, it is negative on the intervals $(-\infty, -2)$ and $(3, \infty)$.

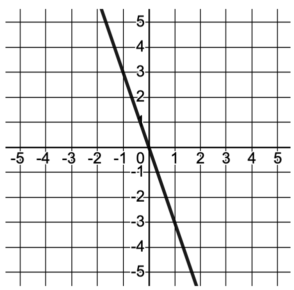


   a. Why are -2 and 3 not included in the negative interval?

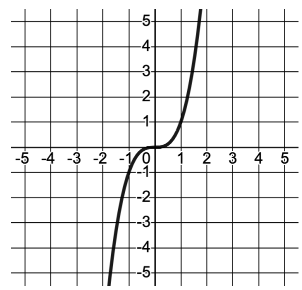
   b. Over what interval(s) is the function positive?

2. Over which interval(s) is each function positive?

   a.

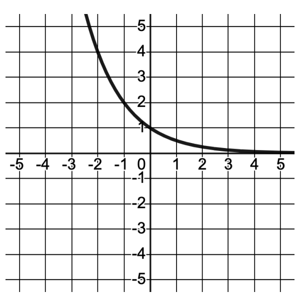


   b.

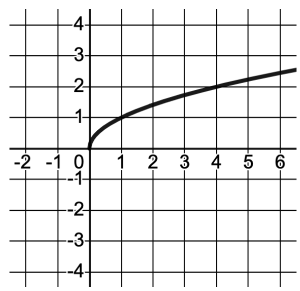


3. Over which interval(s) is each function positive?

   a.

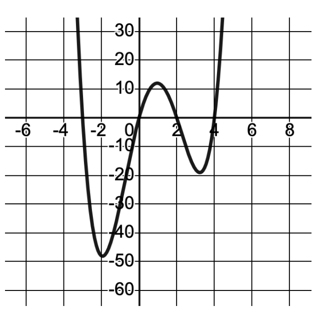


   b.

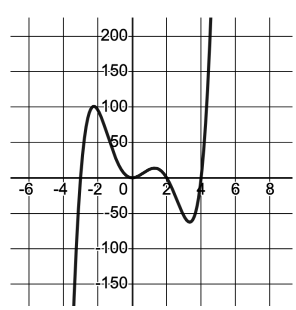


4. Estimate the intervals over which each function is positive.

   a.



   b.



Increasing, Decreasing, Constant

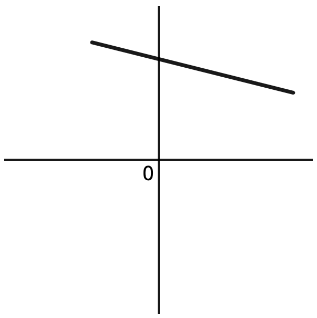
5. Try to fill in the blanks.

a. When a function has a positive slope, the function is increasing. When a function has a negative slope, it is ___________________.

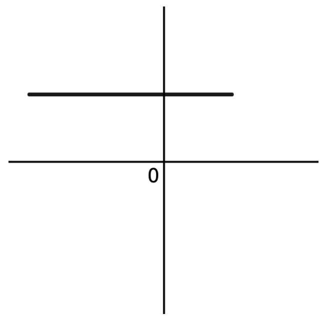
b. When the slope of a function is ____, it is described as constant.

6. Identify each graph below as increasing, decreasing or constant.

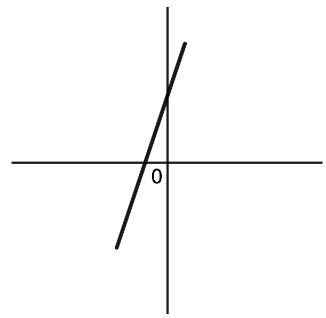
a.



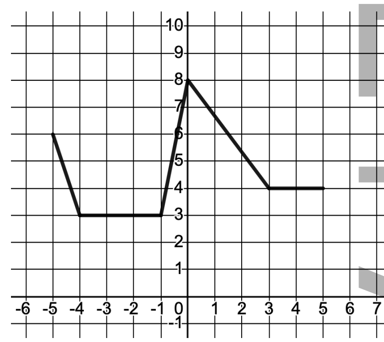
b.



c.



7. The word "constant" is used to describe a section of a graph where the function's *y*-values do not change as the *x*-values increase. The *y*-values are constant. Consider the function shown.



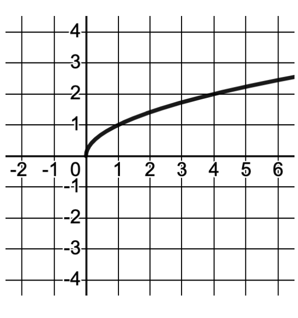
a. Identify all intervals over which the function is constant.

b. Identify the interval(s) over which it is decreasing.

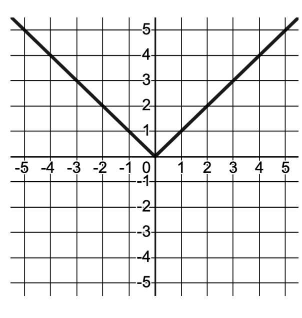
c. Why are the endpoints not included in the increasing, decreasing or constant intervals?

8. Identify all intervals over which the function is increasing.

a.



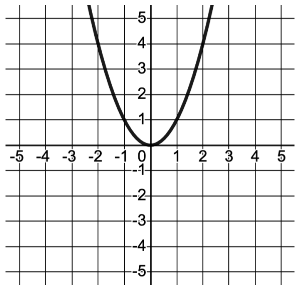
b.



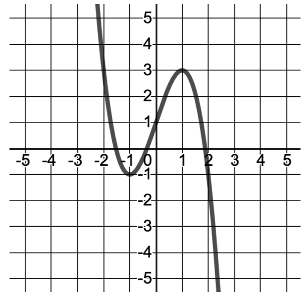
SAMPLE PAGES

9. Identify all decreasing intervals.

a.

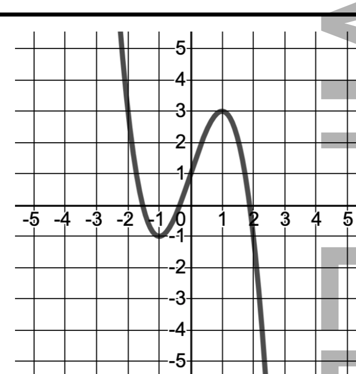


b.



Minima & Maxima

10. When a function **changes** from decreasing to increasing, it has a local (or relative) minimum. When it changes from increasing to decreasing, it has a local (or relative) maximum.

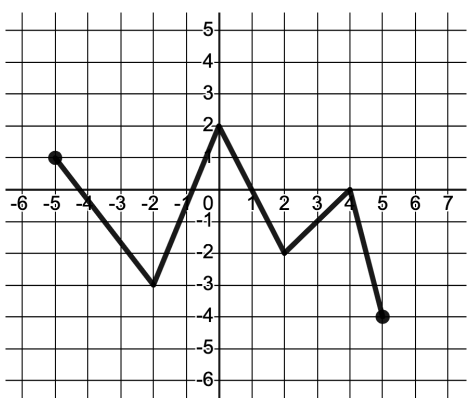


a. Identify the coordinates of the local minimum.

b. Identify the coordinates of the local maximum.

11. Identify all of the local minimums (minima) and maximums (maxima) in each graph.

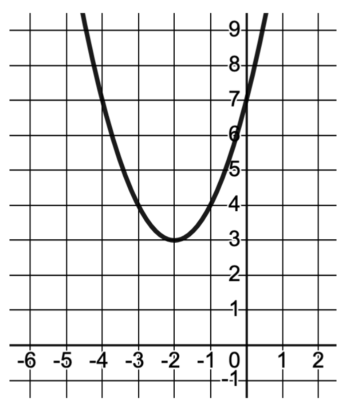
a.



Local minima:

Local maxima:

b.



Local minima:

Local maxima:

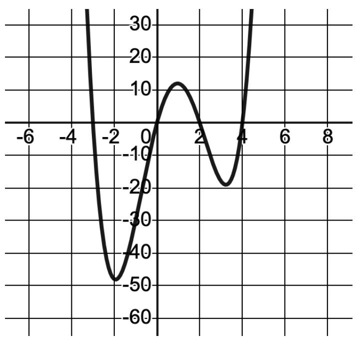
12. The lowest *y*-value of the function is called the global (or absolute) minimum, while the global (or absolute) maximum is the highest *y*-value. Identify the global minimum and maximum for each function in the previous scenario.

a.

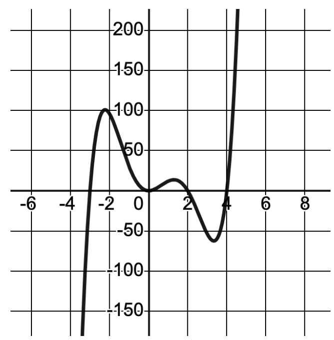
b.

13. Estimate the global minimum and maximum for each function.

a.

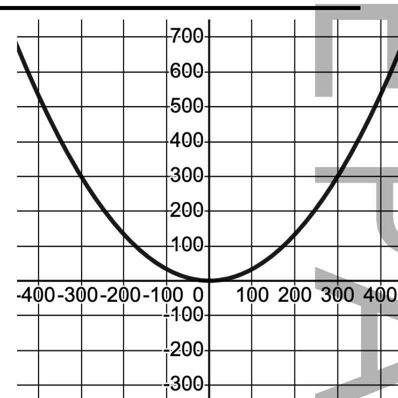


b.



End Behavior

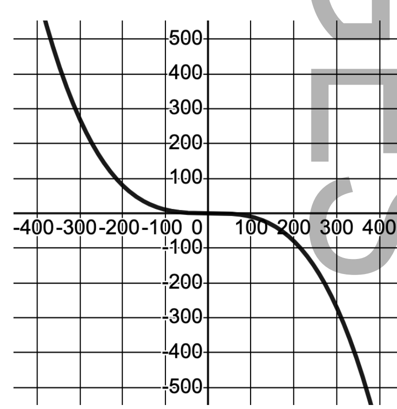
14. As you move far to the right or far to the left on a function's graph, it may approach a specific value or it may approach positive or negative infinity (∞). Consider the function shown. Assume it continues beyond the edges of the graph.



a. As the *x*-values get larger, how do the *y*-values change?

b. How do the *y*-values change as the *x*-values get larger in the negative direction?

15. Consider the function shown. Assume it continues beyond the edges of the graph.



a. As you move to the right on the function (as the *x*-values get closer to positive infinity), what do the *y*-values get closer to?

b. As you move to the left on the function (as the *x*-values approach negative infinity), what do the *y*-values approach?

There is a more concise way to describe a function's end behavior, as shown.

End behavior statement	More concise form
As x approaches infinity, y approaches negative infinity.	As $x \to \infty, y \to -\infty$
As x approaches negative infinity, y approaches infinity.	As $x \to -\infty, y \to \infty$

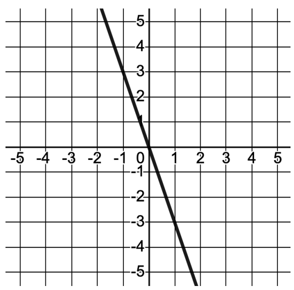
16. Write each statement below in the more concise form.

a. As x approaches infinity, y approaches zero.

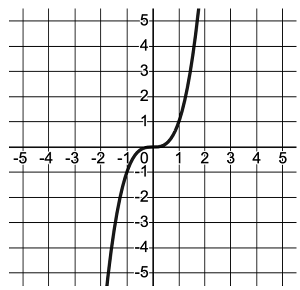
b. As x approaches negative infinity, y approaches three.

17. Identify the end behavior of each function.

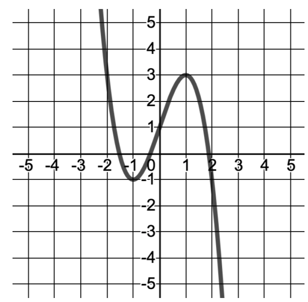
a.



b.



c.

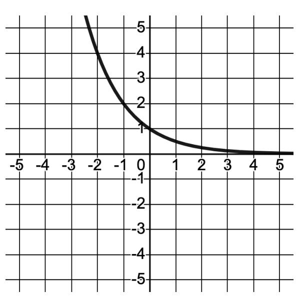


As $x \to \infty$, $y \to$_____

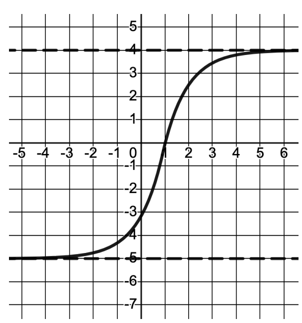
As $x \to -\infty$, $y \to$_____

18. Identify each function's end behavior.

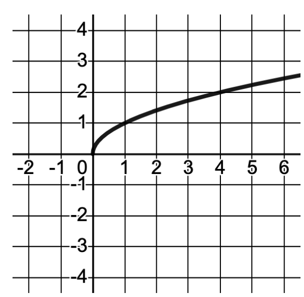
a.



b.



c.

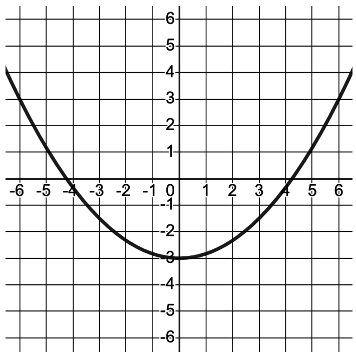


Section 3
Even & Odd Functions

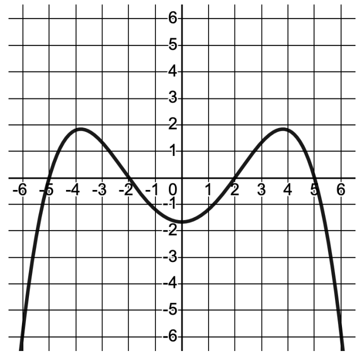
Use this page for taking notes or anything else that helps you learn.

5. Each of the previous parabolas is symmetrical across the y-axis. The shape to the right of the y-axis is mirrored to the left of the y-axis. Since the function $y = x^2$ produces a parabola with this type of symmetry and the exponent in $y = x^2$ is **even**, functions with this symmetry are called **even functions**. Does each graph below show an even function?

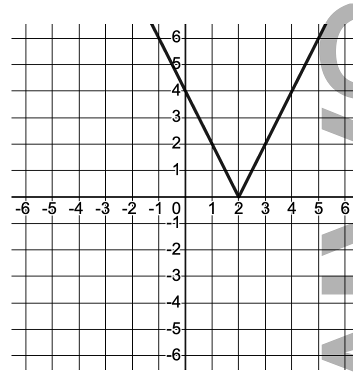
a.



b.

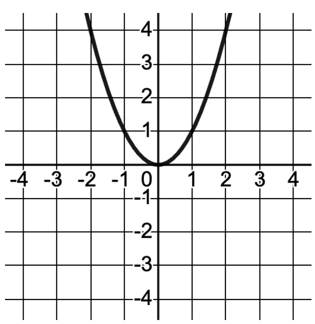


c.

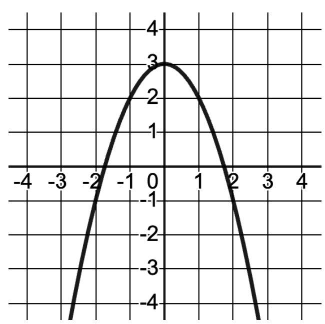


6. To see an even function's symmetry, look at points with opposite x-values. Use the graphs to find each value below it. What do you notice?

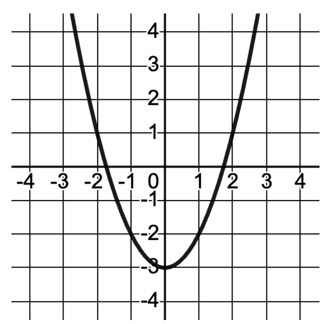
$f(x) = x^2$



$g(x) = -x^2 + 3$



$h(x) = x^2 - 3$



a. $f(2) =$ ____

$f(-2) =$ ____

b. $g(2) =$ ____

$g(-2) =$ ____

c. $h(2) =$ ____

$h(-2) =$ ____

A function is **even** if opposite x-values have equal y-values. For every x-value, $f(-x) = f(x)$. If you have a function's equation, replace x with $-x$ and simplify. If simplifying $f(-x)$ produces the original equation, $f(x)$, the function is **even**.

7. Suppose $f(x) = x^2 + 3$. A function is even if $f(-x) = f(x)$.

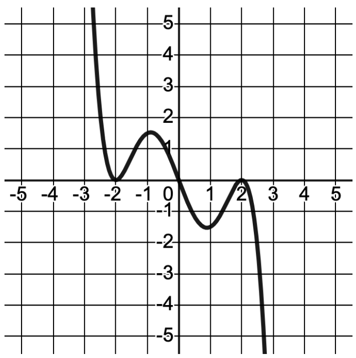
1. Replace x with $-x$. $f(-x) = (-x)^2 + 3 \rightarrow f(-x) = x^2 + 3$

2. Since $f(-x)$ is equivalent to $f(x)$, the original function is ____________.

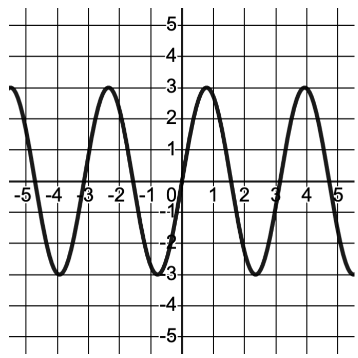
SAMPLE PAGES

12. For each of the previous graphs, the function is symmetrical when rotated around the origin. If you rotate the function 180° around the point (0, 0), it looks the same. Since the graph for the function $y = x^3$ has this type of symmetry and the exponent in x^3 is **odd**, all functions with this type of symmetry are called **odd functions**. Does each graph below show an odd function?

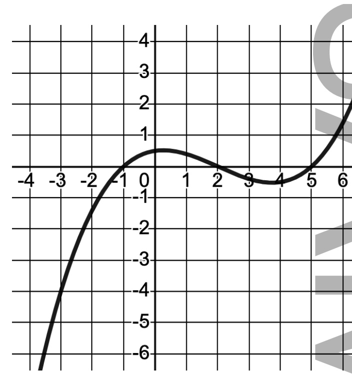
a.



b.

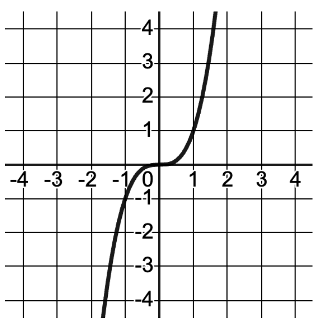


c.

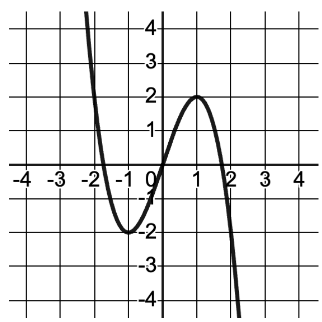


13. To see an odd function's symmetry, look at points with opposite x-values. Use the graphs to find each value below it. What do you notice?

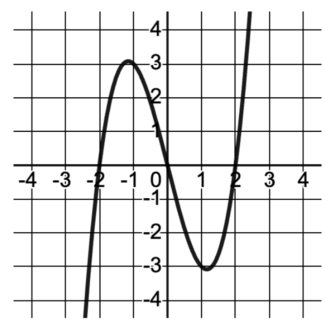
$f(x) = x^3$



$g(x) = -x^3 + 3x$



$h(x) = x^3 - 4x$



a. $f(1) =$ ____

$f(-1) =$ ____

b. $g(1) =$ ____

$g(-1) =$ ____

c. $h(1) =$ ____

$h(-1) =$ ____

A function is **odd** if opposite x-values have opposite y-values. For every x-value, $f(-x) = -f(x)$. If you have a function's equation, replace x with $-x$ and simplify. If simplifying $f(-x)$ produces the opposite of the original function, $-f(x)$, the function is **odd**.

14. For example, suppose $f(x) = x^3 - x$.

1: Replace x with $-x$. $f(-x) = (-x)^3 - (-x) \rightarrow f(-x) = -x^3 + x$

2. Since $f(-x)$ is the <u>opposite</u> of $f(x)$, the original function is ________.

Answer Key

1.	a. $(-1)^2 \rightarrow 1$ b. $(-1)^3 \rightarrow -1$ c. $(-1)^3 - (-1)^2 \rightarrow -1 - 1 \rightarrow -2$
2.	a. $(-x)^2 \rightarrow x^2$ b. $(-x)^3 \rightarrow -x^3$ c. $(-x)^2 - (-x)^3 \rightarrow x^2 + x^3$
3.	a. $-f(x) = -x^2 + 6$ b. $-f(x) = 4x^5 + x^3$ c. $-f(x) = 4x^6 - 5x^4 + 3$
4.	The y-axis is the graph's line of symmetry. The shape on the right side of the y-axis is mirrored on the left side of the y-axis.
5.	a. Yes b. Yes c. No
6.	a. $f(2) = f(-2) = 4$ b. $g(2) = g(-2) = -1$ c. $h(2) = h(-2) = 1$
7.	even
8.	$f(-x) = 7(-x)^4 - 18(-x)^2 \rightarrow 7x^4 - 18x^2$ Since $f(-x) = f(x)$, $f(x)$ is even.
9.	$f(-x) = -3(-x)^2 + 2(-x) + 1$ $\rightarrow -3x^2 - 2x + 1$ Since $f(-x) \neq f(x)$, $f(x)$ is not even.
10.	a. $f(-x) = 3(-x)^4 + 5(-x)^3 - 7$ $\rightarrow 3x^4 - 5x^3 - 7$ Since $f(-x) \neq f(x)$, $f(x)$ is not even. b. $f(-x) = \left((-x)^2 + 6\right)^3 - 4$ $\rightarrow \left(x^2 + 6\right)^3 - 4$ Since $f(-x) = f(x)$, $f(x)$ is even.
11.	The function looks the same if its graph is rotated 180° around the point (0, 0).
12.	a. Yes b. Yes c. No
13.	a. $f(1) = 1$, $f(-1) = -1$ b. $g(1) = 2$, $g(-1) = -2$ c. $h(1) = -3$, $h(-1) = 3$
14.	odd
15.	$f(x) = -7(-x)^3 + 18(-x) \rightarrow 7x^3 - 18x$ Since $f(-x) = -f(x)$, $f(x)$ is odd.
16.	a. $f(-x) = 3(-x) + 1 \rightarrow -3x + 1$ Since $f(-x) \neq -f(x)$, $f(x)$ is not odd. b. $f(-x) = \lvert -x \rvert - 2 \rightarrow \lvert x \rvert - 2$ Since $f(-x) = f(x)$, $f(x)$ is even, not odd.
17.	a. $f(-x) = 54(-x)^5 - 32(-x)^3 + 12(-x)$ $\rightarrow -54x^4 + 32x^3 - 12x$ Since $f(-x) = -f(x)$, $f(x)$ is odd. b. $f(-x) = -7(-x)^6 + 18(-x)^3$ $\rightarrow -7x^6 - 18x^3$ Since $f(-x) \neq -f(x)$ and $f(-x) \neq f(x)$, the function is neither even nor odd.
18.	a. $3(H + 1) \rightarrow 3H + 3$ b. $\left(-2m^3\right)^2 \rightarrow 4m^6$ c. $f\left(x^2\right) \rightarrow 3\left(x^2\right) \rightarrow 3x^2$ d. $g(3x) \rightarrow (3x)^2 \rightarrow 9x^2$
19.	a. $g\left(x^2 + 2\right) \rightarrow \left(x^2 + 2\right) + 6 \rightarrow x^2 + 8$ b. $f(x + 6) \rightarrow (x + 6)^2 + 2$ $\rightarrow (x^2 + 12x + 36) + 2 \rightarrow x^2 + 12x + 38$
20.	a. $f(-1) = -8 \rightarrow g(-8) = 64 - 16 - 4$ $\rightarrow g(-8) = 44 \rightarrow g\left(f(-1)\right) = 44$ b. $g(-3) = 9 - 6 - 4 \rightarrow g(-3) = -1$ $\rightarrow f(-1) = -8 \rightarrow f\left(g(-3)\right) = -8$
21.	a. $f\left(x^2 + 2x - 4\right) \rightarrow 3\left(x^2 + 2x - 4\right) - 5$ $\rightarrow \left(3x^2 + 6x - 12\right) - 5 \rightarrow 3x^2 + 6x - 17$ b. $g(3x - 5) \rightarrow (3x - 5)^2 + 2(3x - 5) - 4$ $\rightarrow (9x^2 - 30x + 25) + (6x - 10) - 4$ $\rightarrow 9x^2 - 24x + 11$
22.	a. $f(1) = 2 \rightarrow g(2) = -3 \rightarrow g\left(f(1)\right) = -3$ b. $g(0) = 3 \rightarrow f(3) = 1 \rightarrow f\left(g(0)\right) = 1$ c. $g(-6) = -3 \rightarrow f(-3) = 4$ $\rightarrow f\left(g(-6)\right) = 4$
23.	a. $f\left(x^2\right) \rightarrow \sqrt{x^2 - 7}$ b. $g\left(\sqrt{x - 7}\right) \rightarrow \left(\sqrt{x - 7}\right)^2 \rightarrow x - 7$
24.	a. $f\left(\frac{1}{x}\right) \rightarrow \frac{\frac{1}{x} - 5}{\frac{1}{x}} \cdot \frac{x}{x} \rightarrow \frac{1 - 5x}{1} \rightarrow 1 - 5x$ b. $g\left(\frac{x - 5}{x}\right) \rightarrow \frac{1}{\frac{x - 5}{x}} \rightarrow 1 \cdot \frac{x}{x - 5} \rightarrow \frac{x}{x - 5}$
25.	$f(x) = \sqrt[3]{x}$, $g(x) = 5x - 2$ or $f(x) = \sqrt[3]{x - 2}$, $g(x) = 5x$
26.	a. $g(x) = \frac{1}{x^2}$, $f(x) = x - 3$ or $g(x) = \frac{1}{x}$, $f(x) = (x - 3)^2$ or $g(x) = x^2$, $f(x) = \frac{1}{x - 3}$
27.	a. $g(x) = \frac{x}{x + 4}$, $f(x) = \lvert x \rvert - 6$ b. $g(x) = \sqrt[4]{x - 4}$, $f(x) = 3x + 9$ or $g(x) = x - 4$, $f(x) = 3\sqrt[4]{x} + 9$ or $g(x) = 3\sqrt[4]{x - 4}$, $f(x) = x + 9$
28.	a. $D(x) = x - 0.25x \rightarrow D(x) = 0.75x$ b. $T(x) = x + 0.08x \rightarrow T(x) = 1.08x$
29.	a. $T\left(D(x)\right) \rightarrow$ The discount is applied first, then the tax. b. $D(70) = 0.75(70) = 52.50$ $\rightarrow T(52.50) = 1.08(52.50) = \$56.70$

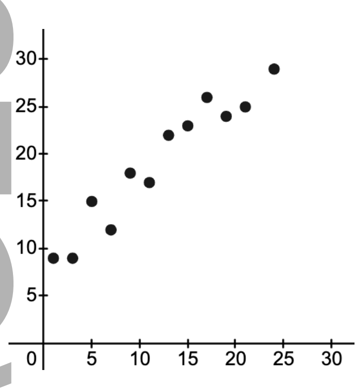
Section 4

Average Rate of Change & Linear Regression

Use this page for taking notes or anything else that helps you learn.

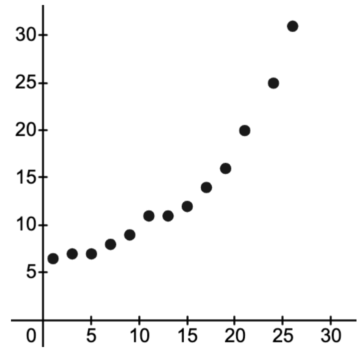
Scatter Plots & Trend Lines

1. As you learn about various functions, you can use them to model patterns in real data sets. Three scatter plots are shown. The first data set follows a linear pattern, but the other two are nonlinear. What words could describe the shapes of the other two data sets?

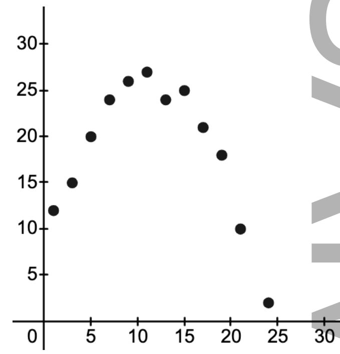


linear

a.



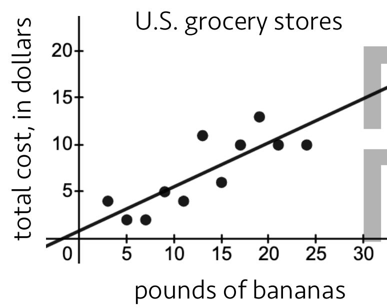
b.



2. When a data set's pattern is linear, a trend line can be drawn to show the data's **average rate of change** (average slope). Consider the data set, trend line and equation shown. Using numbers and units, describe the data's average rate of change.

<u>trend line equation:</u>

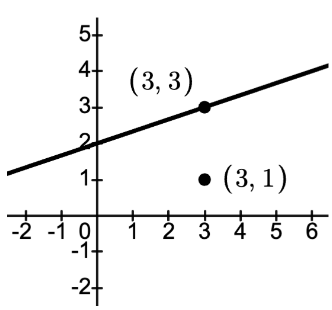
$y = 0.5x + 0.8$



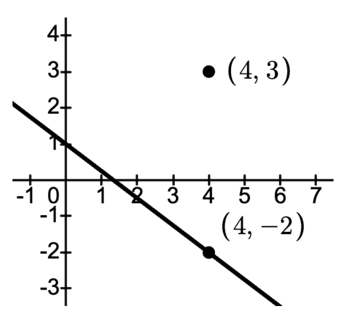
A trend line shows an input's predicted value, while a data point shows the actual value. The vertical distance between the data point and the line is the data point's <u>residual</u>. For example, if a data point is $(1, 5)$ and the trend line predicts $(1, 7)$, the residual is -2. The actual value is 2 below what is predicted. The value of the residual is "actual value – predicted value."

3. Identify the residual of each data point.

a. The residual is ________.



b. The residual is________.



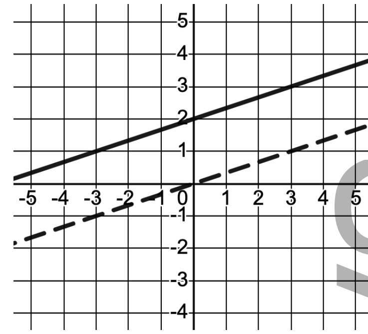
Mini-Unit: Transformations of Functions
These concepts apply to every function in this course.

Section 5
Vertical Transformations

Use this page for taking notes or anything else that helps you learn.

Transformations of Functions

1. The equation for the dashed line is $y = \frac{1}{3}x$.

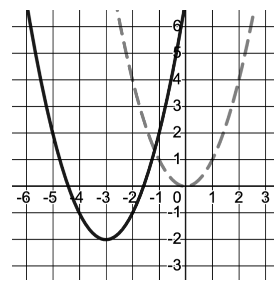


   a. What is the solid line's equation?

   b. What equation forms a line 3 units below $y = \frac{1}{3}x$?

The equation $y = \frac{1}{3}x$ makes a line. Changing the equation to $y = \frac{1}{3}x + \mathbf{2}$ moves the original line **up 2** units. The equation $y = \frac{1}{3}x - \mathbf{3}$ moves the original line **down 3** units. In this lesson, you will learn how changing an equation changes its graph.

2. Two functions are shown in the graph.

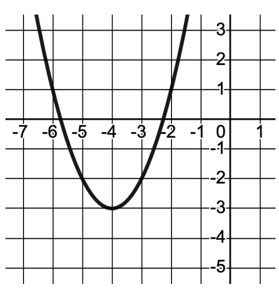


   The dashed curve is the graph of $y = x^2$.

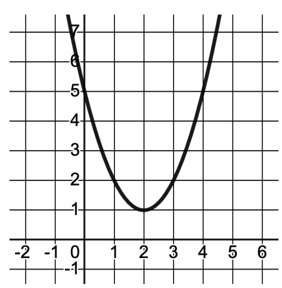
   The solid curve has the same shape as $y = x^2$, but it is moved to the left ___ units and _________ 2 units.

3. The solid curve's equation is $y = (x + 3)^2 - 2$. Though the "$-$ 2" in the equation may look like what you expect, you might wonder why the "+ **3**" moves the parabola **left 3** units. You will learn that later. Using the previous equation as your guide, try to guess the equation for each curve shown.

   a.



   b.



There is more to learn about how changing a function changes its graph. Since the word "transform" means to "change the shape," this topic is called **transformations of functions**. A function can be transformed in 3 ways:

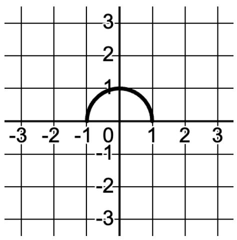
1) shift: move left, right, up or down
2) stretch: shrink or expand in the vertical or horizontal direction
3) reflect: flip across a line or axis

Vertical Transformations

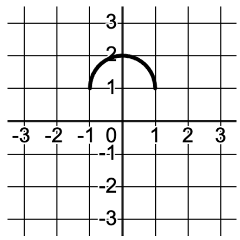
Transformations can be vertical or horizontal. Consider vertical transformations first.

Vertical transformations change a function's outputs (y-values). There are 3 types:

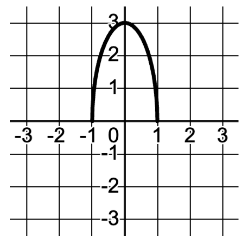
original function: $f(x)$



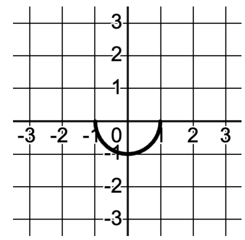
shift: $f(x) + d$



stretch: $a \cdot f(x)$



reflect: $-f(x)$



4. To help you interpret function notation, $f(x)$, remember that x is the input variable and $f(x)$ is the output; $f(x)$ is y. If you rewrite $f(x) + d$ as $y + d$, you can see $f(x) + d$ means add "d" to the y-values. Rewrite each of the 3 transformations by replacing $f(x)$ with y.

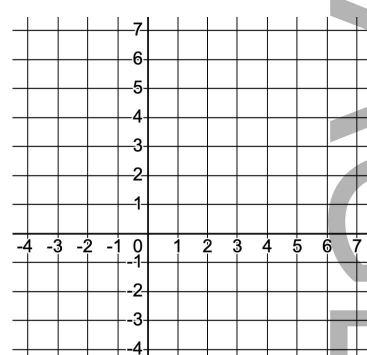
a. $f(x) + d$ b. $a \cdot f(x)$ c. $-f(x)$

5. If $f(x) = \sqrt{x}$, then $f(2)$ is $\sqrt{2}$. If $f(x) = \sqrt{x}$, what expression is represented by each notation below?

a. $f(5)$ b. $f(4x)$ c. $6f(x)$ d. $f(x) - 3$

Vertical Shifts

6. Before you learn how to shift an entire function up or down, you will first move a single point. Consider each function below. In each one, what is the output if $x = 4$? Fill in each blank and then plot each point in the graph.



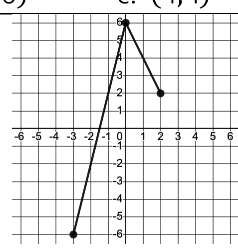
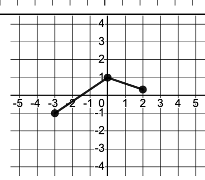
a. $f(x) = \sqrt{x}$ b. $g(x) = \sqrt{x} + 3$ c. $h(x) = \sqrt{x} - 2$

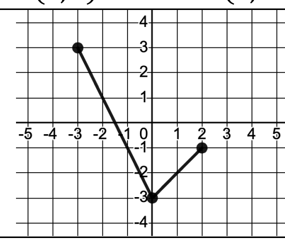
(4 , _____) (4 , _____) (4 , _____)

d. Notice how $g(x)$ moves the $f(x)$ point up _____ units, while $h(x)$ moves the $f(x)$ point down _____ units.

7. To shift a function vertically, move the y-values by the same amount. To move a point up, increase its y-value. To move a point down, decrease its y-value. For example, if $f(x) = x^2$, then $f(x) + 4$ moves every point _________ 4 units, while $f(x) - 7$ moves every point _________ 7 units.

Answer Key

1.	a. $y = \frac{1}{3}x + 2$ b. $y = \frac{1}{3}x - 3$
2.	left 3 units and down 2 units
3.	a. $y = (x+4)^2 - 3$ b. $y = (x-2)^2 + 1$
4.	a. $y + d$ b. $a \cdot y$ c. $-y$
5.	a. $\sqrt{5}$ b. $\sqrt{4x} \rightarrow 2\sqrt{x}$ c. $6\sqrt{x}$ d. $\sqrt{x} - 3$
6.	a. $(4, 2)$ b. $(4, 5)$ c. $(4, 0)$ d. up 3 units.......down 2 units
7.	up 4 units.......down 7 units
8.	a. Line 2 b. Line 1 c. $h(x) = x - 3$; Line 3 d. graph the line $y = x + 2$
9.	a. Y b. X c. $h(x) = x^2 + 2$; Z
10.	a. $g(x) = (x^2 - 2) - 3 \rightarrow x^2 - 5$ b. $g(x) = (\sqrt{x} + 5) + 7 \rightarrow \sqrt{x} + 12$ c. $g(x) = (3^x + 2) - 6 \rightarrow 3^x - 4$
11.	a. y-value b. y-value; 3 c. move the 3 marked points down 3 units and connect the points with segments
12.	a. $(4, 2)$ b. $(4, 6)$ c. $(4, 1)$
13.	 Multiply y-values by 2
14.	 Multiply y-values by $\frac{1}{3}$

15.	a. Y b. $g(x) = 3x^2$; Z c. $h(x) = \frac{1}{2}x^2$; X
16.	a. $\frac{1}{2}(x^2 - 6) \rightarrow \frac{1}{2}x^2 - 3$ b. $3(\sqrt{x} + 5) \rightarrow 3\sqrt{x} + 15$ c. $5(3^x - 2) \rightarrow 5 \cdot 3^x - 10$ Notice how the constant term also changes.
17.	a. $(4, 2)$ b. $(4, -2)$
18.	
19.	1. Multiply y-values by -1 2. Shift up 2 units
20.	For each point, multiply the y-value by -1 and then move the point up 3 units.
21.	a. $y = -\sqrt{x}$ b. $y = -x^2$
22.	a. $y = -x^2 + 6$ b. $y = -\sqrt{x} - 5$ c. $y = -3^x + 2$ Notice how the constant term also changes.
23.	a. $y = x^2 + 2$ b. $y = \frac{1}{2}x^2$ c. $y = -4x^2$ d. $y = 2x^2 + 5$
24.	a. multiply the y-value by -5 and then shift the point up 1 unit b. multiply the y-value by $\frac{2}{3}$ and then shift the point down 6 units

Mini-Unit: Transformations of Functions

These concepts apply to every function in this course.

Section 6

Horizontal Transformations

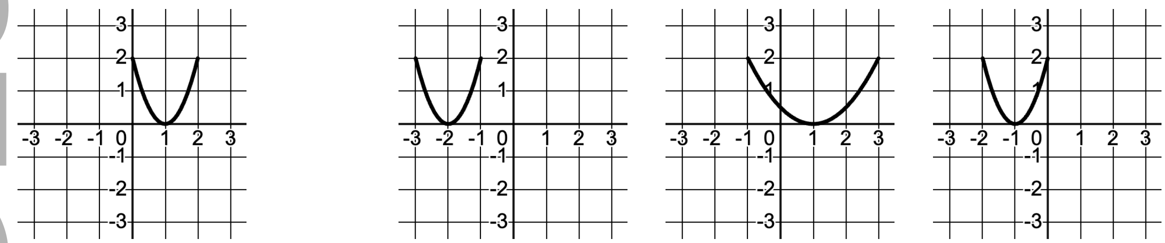
Use this page for taking notes or anything else that helps you learn.

Horizontal Transformations

While vertical transformations change outputs (y-values), horizontal transformations change inputs. They change the x-value before it is put into the function.

There are 3 types of horizontal transformations.

original function: $f(x)$ | shift: $f(x+c)$ | stretch: $f(bx)$ | reflect: $f(-x)$



1. If $f(x) = \sqrt{x}$, what expression is represented by each function notation below?

   a. $f(x-2)$ b. $f(x)-2$ c. $f(x-6)+1$ d. $2f(x)-5$

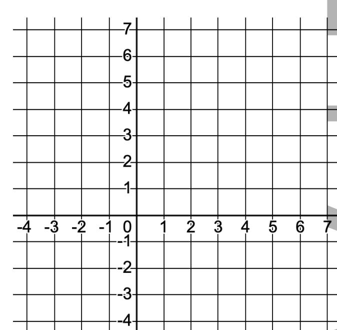
Horizontal Shifts

To understand horizontal shifts, see how a single point moves when you change its function. Horizontal transformations change the x-values before they are put into the function.

2. Consider each function below. What x-value produces an output of 2? Fill in each blank and then plot each point in the graph.

   a. $f(x) = \sqrt{x}$ b. $g(x) = \sqrt{x+3}$ c. $h(x) = \sqrt{x-2}$

   (____ , 2) (____ , 2) (____ , 2)



The function $f(x) = \sqrt{x}$ has a point at (4, 2), while $g(x) = \sqrt{x+3}$ is 3 units to the left of $f(x)$ at (1, 2). The function $h(x) = \sqrt{x-2}$ is 2 units to the right of $f(x)$ at (6, 2).

3. Consider a different function, $f(x) = x^2$. One point on the function is (3, 9). If the function is changed to $g(x) = (x-2)^2$, what input produces an output of 9?

4. Horizontal shifts seem to do the opposite of what you expect. For example, if $f(x) = x^2$, then $f(x+4)$ moves all the points left 4 units, while $f(x-7)$ moves all the points ________ 7 units.

5. The transformation $f(x+5)$ moves the graph of $f(x)$ ____ units to the ___________.

6. The transformation $f(x-6)$ moves the graph of $f(x)$ ____ units to the ___________.

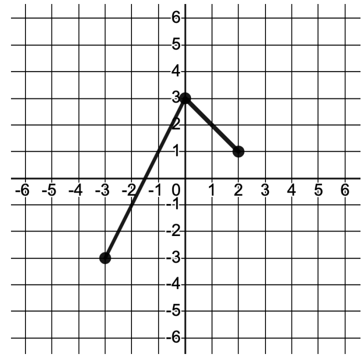
7. For each function shown below, write a new equation that moves it left 10 units.

a. $y = x^2$

b. $y = |x|$

8. The function $f(x)$ is shown in the graph.

a. The function $f(x-4)$ changes $f(x)$ by moving the graph ____ units to the __________.



On the same grid, draw both graphs.

b. $f(x-4)$

c. $f(x+3)$

9. For each function, write the new equation formed by the transformation shown.

a. $f(x) = 3^x$

$g(x) = f(x-2)$

$g(x) =$

b. $f(x) = x^2 - 2$

$g(x) = f(x+7)$

$g(x) =$

c. $f(x) = \sqrt{x} + 5$

$g(x) = f(x-1)$

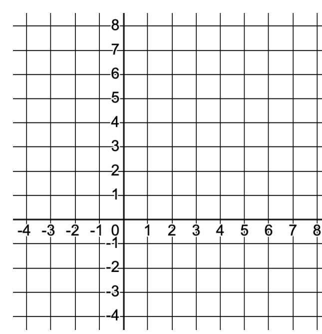
$g(x) =$

Horizontal Shifts
The transformation $f(x+c)$ moves $f(x)$ left c units, while $f(x-c)$ moves $f(x)$ right c units.

Horizontal Stretches

Like horizontal shifts, horizontal stretches do the opposite of what you expect.

10. Consider each function below. In each one, what x-value produces an output of 2? Fill in each blank and then plot each point in the graph.



a. $f(x) = \sqrt{x}$

(____ , 2)

b. $g(x) = \sqrt{2x}$

(____ , 2)

c. $h(x) = \sqrt{\frac{1}{3}x}$

(____ , 2)

The function $f(x) = \sqrt{x}$ has a point at (4, 2), while $g(x) = \sqrt{2x}$ multiplies the x by one-half, moving (4, 2) to (2, 2). For $h(x) = \sqrt{\frac{1}{3}x}$, the x-value is multiplied by 3, moving (4, 2) to (12, 2).

CONTENTS

Unit 2

Absolute Value Functions

SAMPLE PAGES

Section 1
Absolute Value Functions

Use this page for taking notes or anything else that helps you learn.

The Absolute Value Function

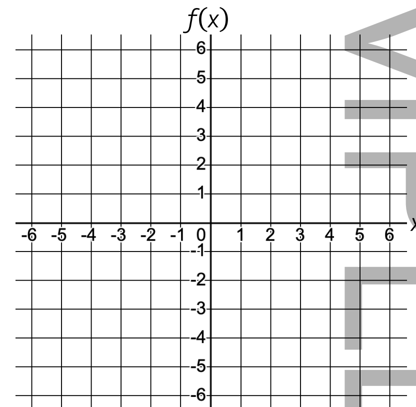
1. To help you learn how to graph an absolute value function, it will help to review what you know about absolute value expressions. As a reminder, the absolute value of a number is how far it is from 0. The absolute value of a number is always positive. Simplify each expression below.

a. $|4|$ b. $|-7|$ c. $|5-3|$ d. $|2-6|$

2. Consider the simplest absolute value function, shown below. Graph the function by using the T-chart to find specific points.

$f(x) = |x|$

x	$f(x)$
−3	
−2	
−1	
0	
1	
2	
3	

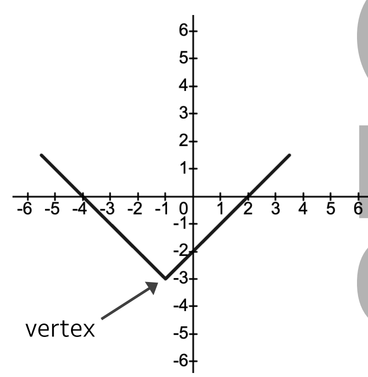


3. An absolute value function has a V-shape formed by drawing 2 separate rays with different domains but the same endpoint. For the function $f(x) = |x|$, find the equations of the two rays that form the V and state the domain of each ray.

4. Each absolute value function has a vertex.

a. Identify the coordinates of the vertex shown.

b. Find the equations of both rays that form the graph of the function shown and state the domain of each ray.



Applying Transformations to Absolute Value Functions

5. When you graph the function $f(x) = |x|$, the vertex of the V is at (0, 0). Use what you have learned about transformations to identify the vertex for each function shown.

a. $f(x) = |x| + 2$

b. $f(x) = |x - 3|$

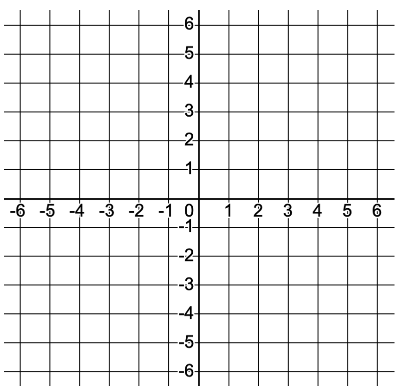
c. $f(x) = |x + 7| + 4$

d. $f(x) = \frac{1}{2}|x - 6| - 1$

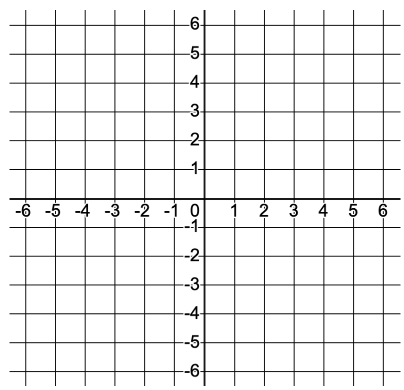
6. What are the coordinates of the function's vertex if its equation is $f(x) = a|x - h| + k$?

7. Use what you have learned about transformations to graph each function.

a. $f(x) = |x| + 2$

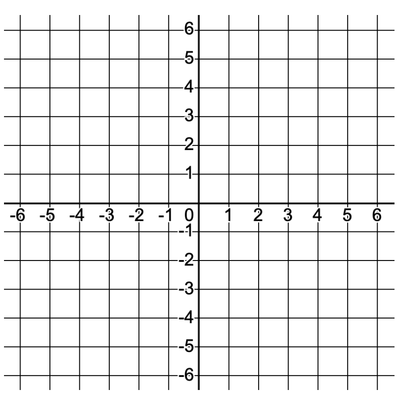


b. $f(x) = |x + 3| - 1$

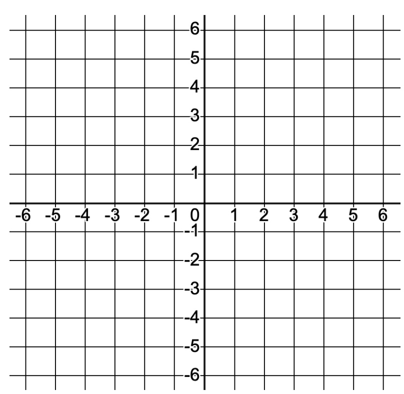


8. In the function $y = a|x|$, the slopes of the rays are $\pm a$. Try to graph each function shown.

a. $y = \frac{1}{2}|x|$

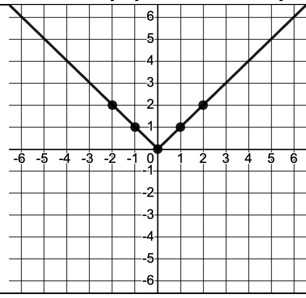
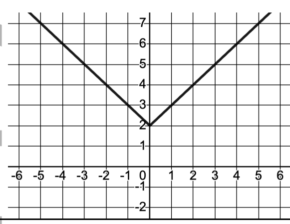
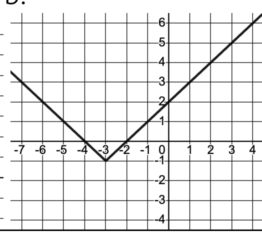
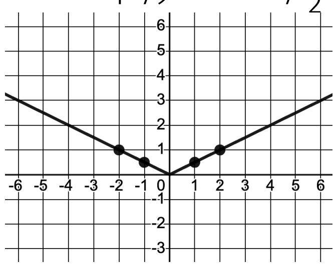
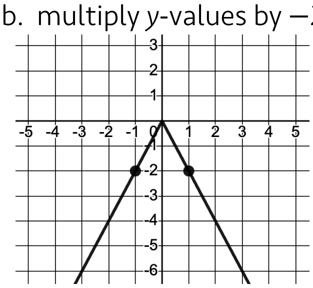
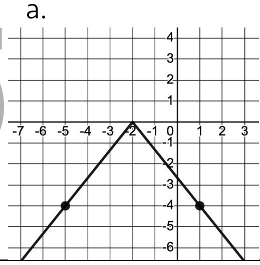
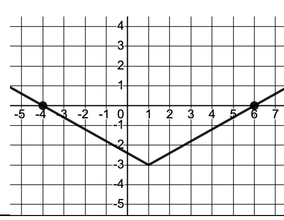
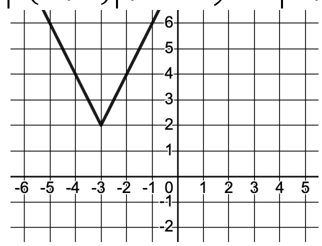
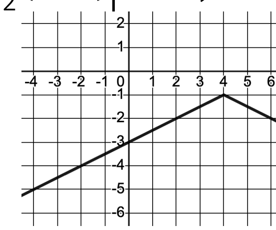
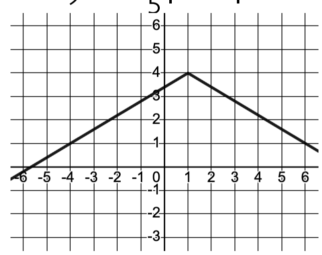
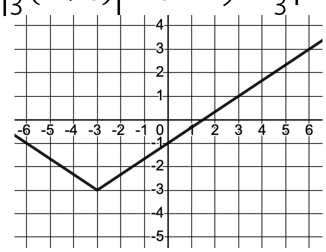


b. $y = -2|x|$



The Vertex Form for an absolute function is $f(x) = a|x - h| + k$. When the function is in this form, the coordinates of the vertex are located at (h, k). The slopes of the rays are $\pm a$.

Answer Key

1.	a. 4 b. 7 c. $\lvert 2\rvert \rightarrow 2$ d. $\lvert -4\rvert \rightarrow 4$
2.	
3.	$y = \begin{cases} x & \text{domain: } x \geq 0 \\ -x & \text{domain: } x < 0 \end{cases}$
4.	a. $(-1, -3)$ b. $y = \begin{cases} x-2; & x \geq -1 \\ -x-4; & x < -1 \end{cases}$
5.	a. $(0, 2)$ b. $(3, 0)$ c. $(-7, 4)$ d. $(6, -1)$
6.	(h, k)
7.	a.  b. 
8.	a. multiply y-values by $\frac{1}{2}$  b. multiply y-values by -2 
9.	a.  b. 
10.	Factor out a GCF of 4 to rewrite the function as $y = \lvert 4(x-1)\rvert + 3$. This shows a shift to the right 1, a horizontal compression by a factor of 4, and a shift up 3.
11.	a. $y = \lvert 2(x+4)\rvert - 3$ vertex: $(-4, -3)$ b. $y = \lvert 3(x-7)\rvert + 6$ vertex: $(7, 6)$
12.	a. $y = 2\lvert x+4\rvert - 3$ b. $y = 3\lvert x-7\rvert + 6$ c. $y = \lvert 5(x+2)\rvert - 7 \rightarrow y = 5\lvert x+2\rvert - 7$
13.	a. $y = \lvert -2\rvert \cdot \lvert x\rvert + 1 = 2\lvert x\rvert + 1$ b. $y = \lvert -4x+2\rvert = \lvert -4(x-0.5)\rvert$ $\rightarrow y = 4\lvert x-0.5\rvert$ c. $y = \left\lvert -6\left(x+\frac{4}{3}\right)\right\rvert + 5 = 6\left\lvert x+\frac{4}{3}\right\rvert + 5$
14.	a. $y = \lvert 2(x+3)\rvert + 2 \rightarrow y = 2\lvert x+3\rvert + 2$  b. $y = -\left\lvert \frac{1}{2}(x-4)\right\rvert - 1 \rightarrow y = -\frac{1}{2}\lvert x-4\rvert - 1$ 
15.	a. $y = -\frac{3}{5}\lvert x-1\rvert + 4$  b. $y = \left\lvert \frac{2}{3}(x+3)\right\rvert - 3 \rightarrow y = \frac{2}{3}\lvert x+3\rvert - 3$ 
16.	a. $y = 2\lvert x-1\rvert - 4$ b. $y = -\frac{4}{3}\lvert x+2\rvert + 5$

Section 2
Solving Absolute Value Equations

Use this page for taking notes or anything else that helps you learn.

Solving Absolute Value Equations Algebraically

You can use what you know about absolute functions to solve absolute value equations.

1. Consider the equations shown below. List all x-values that make this equation true.

a. $|x| = 3$ b. $|x| = 0$ c. $|x| = -3$

2. The previous equation in part a. has two solutions: 3 and -3. Another type of equation with 2 solutions is $x^2 = 9$. Like quadratic equations, absolute value equations can also have two solutions. Consider each equation below. Try to solve each equation by guessing and checking different numbers.

a. $|x| = 7$ b. $|x| + 1 = 6$ c. $|x + 1| = 2$

3. As absolute value equations get more complicated, it will help to use an algebraic approach to solve each equation. Absolute value equations can be split into 2 cases, the positive case and the negative case. For example, if $|x| = 7$, then x is either 7 or -7. Consider each statement below and try to fill in the blank for each equation.

a. If $|x + 3| = 8$, then $x + 3 =$_____ or $x + 3 =$_____.

b. If $|3 + 2x| = 4$, then $3 + 2x =$_____ or $3 + 2x =$_____.

4. As you see in the previous scenario, absolute value equations can be split into two cases. Solve each equation below by splitting it into two cases and solving each case.

a. $|x - 2| = 7$

$x - 2 = 7$ $x - 2 = -7$

$x =$ ____ $x =$ ____

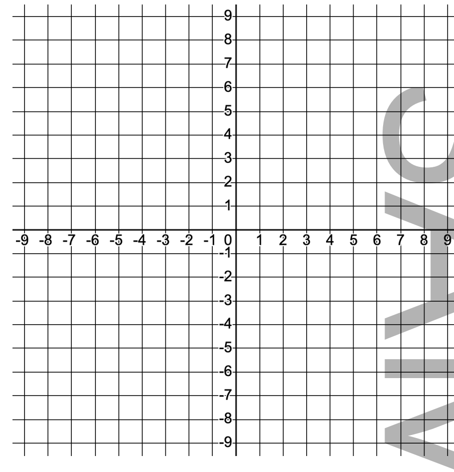
$x =$ ____ or ____

b. $|2x - 1| = 15$

Solving Absolute Value Equations Graphically

In addition to solving absolute value equations algebraically, you can also solve them graphically.

9. Consider the equation below. To solve it graphically, make each side of the equation a separate function. Graph them and find the x-values at the points where they intersect.



$|x-2|-3=4$

a. left function:

$y =$

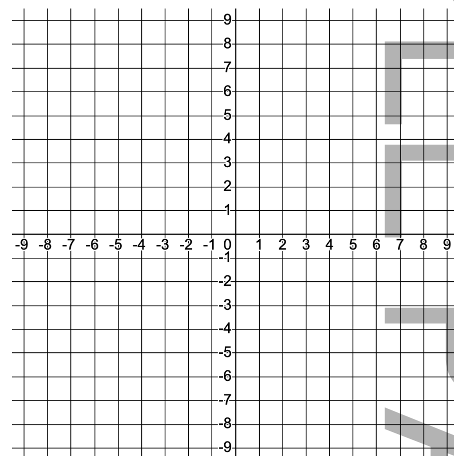
b. right function:

$y =$

c. solution:

$x =$

10. On the coordinate plane provided, graph the following functions:

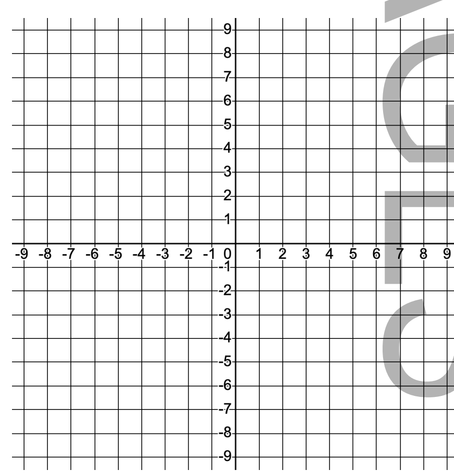


$f(x) = -|x+3|+5$

$g(x) = 2$

11. At what x-values do the functions intersect each other. Where does $f(x) = g(x)$?

12. Graph $f(x) = 3|x-1|-5$ and $g(x) = -x+4$ on the same plane. Using your graph, state the solutions for the equation below.



$3|x-1|-5 = -x+4$

Section 3

Solving Absolute Value Inequalities

Use this page for taking notes or anything else that helps you learn.

6. It can take a long time to draw a detailed number line, so it is easier to draw number lines like the ones shown below. Write the inequality that is represented on each simpler number line.

a.

b.

7. Show the numbers that satisfy the inequality by drawing and shading a simple number line.

a. $x > 6$

b. $x \leq -1.5$

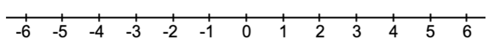
Solving Absolute Value Inequalities Algebraically

8. Now that you have solved absolute value equations and reviewed inequalities, you can combine these two concepts to solve an absolute value inequality. Consider the inequality below.

$|x| \leq 2$

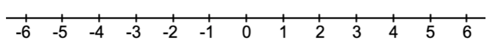
a. List all integers that make the inequality true.

b. On the number line below, show all x-values that make this inequality true.



9. Graph the inequality's solution on the number line and write the solution using interval notation.

$|x| \geq 4$



interval notation:

10. In the previous scenario, notice how the solution includes numbers greater than 4 or less than −4. Show the solution to each inequality using interval notation and also on a number line.

a. $|x| < 5$

b. $|x| > 3$

11. Like absolute value equations, inequalities are solved by **isolating** the absolute value expression, then splitting it into 2 cases. In part a. above, for example, the 2 cases are "less than 5 and greater than −5. Solve each inequality below and show each solution on a number line.

a. $|x - 3| \geq 6$

$x - 3 \geq 6$ $\quad$ $x - 3 \leq -6$

b. $|x + 2| - 1 < 3$

SAMPLE PAGES