SUMMIT MATH

Learn at your **OWN** pace.

ADVANCED ALGEBRA & TRIGONOMETRY SERIES

BOOK 3

UNIT 5: POLYNOMIAL FUNCTIONS & ROOTS

UNIT 6: INVERSE, SQUARE ROOT & CUBE ROOT FUNCTIONS

INTRODUCTION

Learning math through Guided Discovery:
Guided Discovery is designed to help you experience a sense of discovery as you learn each topic.

Why this curriculum series is named Summit Math:
Learning through Guided Discovery is like hiking. Learning and hiking both require effort and persistence and people naturally move at different paces, but they can reach the summit if they keep moving forward. Whether you race rapidly through the book or step slowly through each scenario, this textbook is designed to keep advancing your learning until you reach the summit.

Guided Discovery Scenarios:
The Guided Discovery Scenarios in this book are written and arranged to show you that new math concepts are related to previous concepts you already know. Try each scenario on your own first, check your answer when you finish, and then fix any mistakes, if needed. Making mistakes and struggling are essential parts of the learning process.

Homework & Extra Practice Scenarios:
After you complete the scenarios in each Guided Discovery section, extra practice will help you develop better memory of each topic. Use the Homework & Extra Practice Scenarios to improve your understanding and to increase your retention.

The Answer Key:
The Answer Key is included to give you teacher-like guidance. When you finish a scenario, you can get immediate feedback. If the Answer Key does not help you fully understand the scenario, try to get additional guidance from another student or a teacher.

Find more resources at:
www.summitmath.com

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GUIDED DISCOVERY SCENARIOS

The Guided Discovery Scenarios are designed to help you experience a sense of discovery as you learn. Explanations are brief and carefully timed to allow you to build your learning at a comfortable pace. The Answer Key supports your learning by giving you immediate feedback and helping you understand each step before moving on.

As you complete each scenario, follow these steps.

Step 1: Try the scenario.
Read the scenario carefully and try to work through it. Be willing to struggle.

Step 2: Check the Answer Key.
The Answer Key can guide you through the scenario, show you new ideas, or help you find mistakes in your steps.

Step 3: Fix your mistakes, if needed.
Mistakes are learning opportunities. If your result does not match the Answer Key, check your work and look for errors. If you still need guidance, seek help from a classmate or teacher.

When you are ready, try the next scenario and repeat this cycle.

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Unit 5

Cubic & Polynomial Functions

Section 1

Cubic Functions

Use this page for taking notes or anything else that helps you learn.

Introduction to Cubic Functions

1. The graph of the function $y = x^3$ is shown. To see why it has this shape, calculate the missing y-values in the table shown and plot these points in the graph.

   a. Identify the function's end behavior.

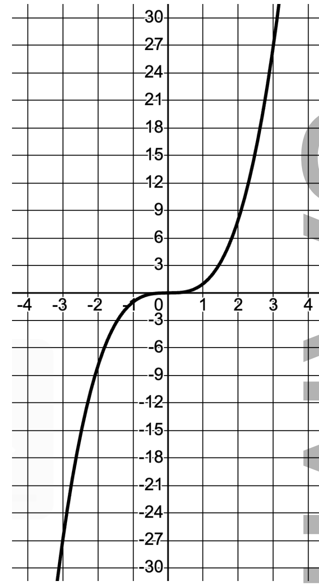
      As $x \to \infty$, $y \to$

      As $x \to -\infty$,

   b. Over what interval(s) is it increasing?

   c. Is the function even, odd, or neither? Explain.

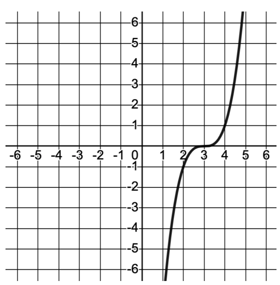
x	y
-3	
-2	
-1	
0	
1	
2	
3	



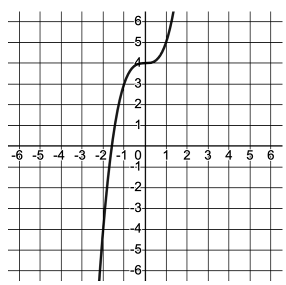
Applying Transformations to Cubic Functions

2. Each graph shows a transformation of $y = x^3$. Describe the transformation using the following words: horizontal, vertical, shift, stretch, compression, reflection.

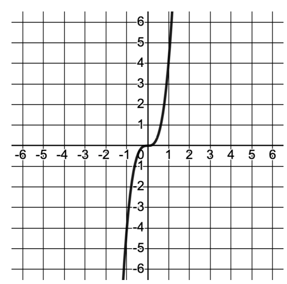
   a.



   b.



   c.



3. If $f(x) = x^3$, write the function $g(x)$ produced by the transformation(s). The first one is done for you.

   a. shift down 5 units

      $g(x) = x^3 - 5$

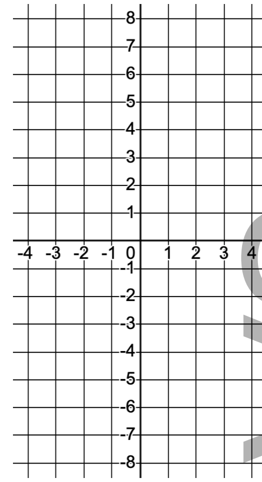
   b. shift up 4 units, right 2 units

   c. reflect across x-axis, shift left 1 unit

   d. stretch vertically by a factor of 4

SAMPLE PAGES

4. Draw a simple sketch of $y = x^3$ again. Use the function to find the coordinates of 5 points and then plot those points before you draw the curve.



$(-2,\_\_\_\_\_)$

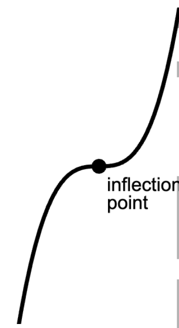
$(-1,\_\_\_\_\_)$

$(0,\_\_\_\_\_)$

$(1,\_\_\_\_\_)$

$(2,\_\_\_\_\_)$

5. On the graph of $y = x^3$, the point $(0, 0)$ is called the inflection point. It is the point where the function's curvature changes. Use what you know about transformations to identify the coordinates of the inflection point on each function below.



a. $f(x) = (x + 4)^3$ b. $g(x) = (x - 2)^3 - 6$

6. Use "order of operations" to determine the inflection point on each function.

a. $f(x) = 3x^3 + 3$ b. $g(x) = \frac{1}{3}(x + 7)^3$ c. $h(x) = a(x - h)^3 + k$

7. The next important points on the graph of $y = x^3$ are $(1, 1)$ and $(-1, -1)$. When you transform $y = x^3$, these 2 points move. Apply what you have learned about transformations to identify where these 2 points move to on each function described below.

a. $f(x) = (x + 4)^3$ b. $g(x) = x^3 - 3$ c. $h(x) = (x - 2)^3 - 6$

$(1, 1) \rightarrow$ $(1, 1) \rightarrow$ $(1, 1) \rightarrow$

$(-1, -1) \rightarrow$ $(-1, -1) \rightarrow$ $(-1, -1) \rightarrow$

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Section 2
Polynomial Functions

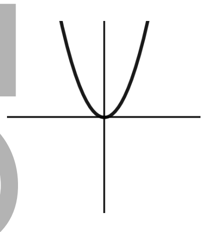
Use this page for taking notes or anything else that helps you learn.

Graphing Polynomial Functions

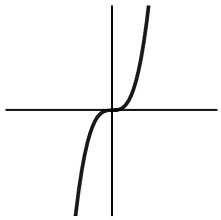
A quadratic function, $y = x^2$, is a 2nd degree function, while a cubic function, $y = x^3$, is a 3rd degree function. You can use what you know about 2nd and 3rd degree **polynomial functions** to learn about the shapes of higher degree polynomial functions (4th degree, 5th degree, etc...).

1. Examine the following functions and look for patterns.

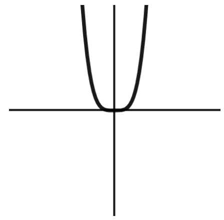
$y = x^2$



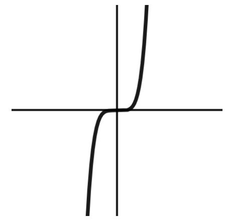
$y = x^3$



$y = x^4$



$y = x^5$



Think about how negative numbers are affected by odd and even exponents as you analyze the functions above. Try to draw a simple sketch of $y = x^6$ and $y = x^7$ and then answer the 2 questions.

a. $y = x^6$

b. $y = x^7$

c. Why do x^2, x^4 and x^6 have similar shapes?

d. Why do x^3, x^5 and x^7 have similar shapes?

2. Using what you have learned about transformations, try to draw a simple sketch of each function.

a. $y = -x^4$

b. $y = -x^5$

c. $y = -x^6$

d. $y = -x^7$

3. Describe the end behavior of each function.

a. $y = x^5$

as $x \to \infty$, $y \to$

as $x \to -\infty$,

b. $y = -x^8$

as $x \to$

as x

c. $y = -x^9$

as

The end behavior of a polynomial function is like $y = x^2$ if the degree is even and like $y = x^3$ if the degree is odd. If the leading coefficient is negative, the end behavior is reflected across the x-axis.

SAMPLE PAGES

The Roots of a Polynomial Function

Polynomial functions have many features. In this section, you will learn about the relationship between a polynomial's equation and its x-intercepts.

4. If you have a function's equation, how do you find its x-intercept(s)? For example, suppose the function is $y = x - 4$. How do you find this function's x-intercept?

5. Each function below is written in factored form, which makes it easier to find the x-intercepts. Identify the x-intercepts of each function.

   a. $y = (x + 3)(x - 5)$ b. $y = x(x + 2)$ c. $y = (x - 1)(x + 6)(x - 8)$

6. A function has x-intercepts at $x = -7$ and $x = 1$. If the leading coefficient of the function is 1, write the function in factored form.

7. Write the polynomial function in factored form, given its x-intercepts.

   a. x-intercepts: $x = 2$, 3 and -4 b. x-intercepts: $x = 0$ and ± 5

8. Identify the x-intercept(s) of each function.

   a. $y = 3\left(x^2 - 4\right)$ b. $y = 2x(x - 3)^2$

9. Identify the x-intercepts of each function.

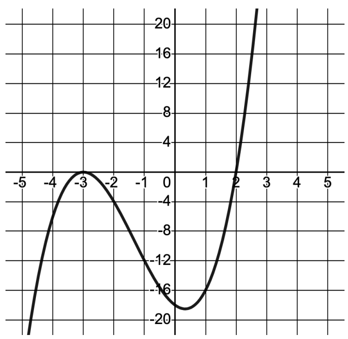
   a. $y = x^2(x + 5.5)$ b. $y = (x + 4)(x - 7)\left(x^2 - 1\right)$

The Shape of a Polynomial Function at its Roots

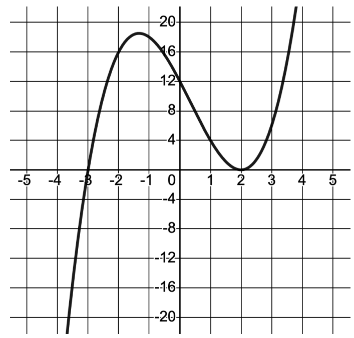
When a quadratic function in factored form is $y = a(x - h)^2$, it touches the x-axis at "h" but does not cross the x-axis. The sign of the y-value is the same on both sides of the root. This behavior also occurs for higher degree polynomial functions. If a factor can be written as $(x - h)^2$, the function touches the x-axis at "h" but does not cross the x-axis.

14. Each function shown has two roots: $(-3, 0)$ and $(2, 0)$. What do you notice when you look at each equation and the shape of the function's graph at its roots?

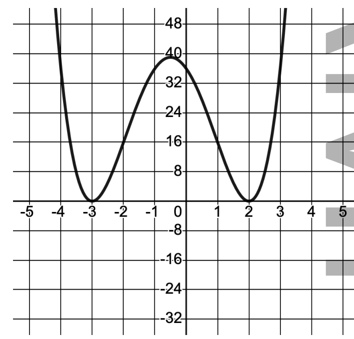
$y = (x - 2)(x + 3)^2$



$y = (x - 2)^2(x + 3)$



$y = (x - 2)^2(x + 3)^2$



15. Will each function cross the x-axis or "bounce" at $x = 3$?

a. $y = 3(x + 5)(x - 3)$

b. $y = -(x - 3)^2$

c. $y = x(x^2 - 9)$

16. Try to draw a simple sketch of each function.

a. $y = (x + 3)^2(x - 2)$

b. $y = -(x + 2)^2(x - 4)$

Section 3
Finding Real Roots of Polynomials

Use this page for taking notes or anything else that helps you learn.

Making Connections Between Roots & Factors

1. A 2nd degree polynomial function has only 2 roots: 3 and -7.

   a. Write the function in factored form.

   b. Convert the function to Standard Form.

2. If you know a polynomial's roots, you can write the function in factored form. Similarly, if you know a polynomial's factors, you can identify its roots.

   a. If a root is 3, the factor is $(x - \_\_\_)$.

   b. If a root is A, the factor is $(x - \_\_\_)$.

   c. If a root is $\sqrt{5}$, what is the factor?

   d. If a root is $\left(2 + \sqrt{6}\right)$, what is the factor?

3. A quadratic function has only 2 roots: $3 + \sqrt{5}$ and $3 - \sqrt{5}$.

   a. Write the function in factored form.

   b. Write the function in Standard Form.

4. Notice the roots of the previous function: $3 \pm \sqrt{5}$. Consider the function $y = x^2 - 6x + 13$.

   a. One of its roots is $3 + 2i$. Guess the other root: ______________

   b. Use the Quadratic Formula or Completing the Square to find the roots of $y = x^2 - 6x + 13$.

5. When you find a quadratic function's roots with the Quadratic Formula or by Completing the Square, they are $\mathrm{A} + \sqrt{\mathrm{B}}$ and $\mathrm{A} - \sqrt{\mathrm{B}}$, unless B is 0. These irrational and/or imaginary roots come in pairs.

   a. If one root is $4 - \sqrt{3}$, the other root is _______________.

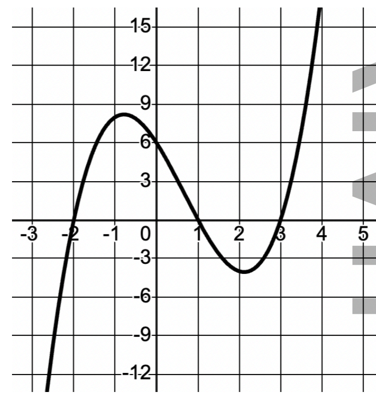
   b. If one root is $1 + 6i$, the other root is _______________.

   c. If one root is $7 - i\sqrt{5}$, what is the other root?

6. Two of a polynomial's roots are 7 and $5-\sqrt{3}$. Write the polynomial in factored form.

7. Write a polynomial in factored form if it has one root at $4i$ and another root at $1+2\sqrt{7}$.

8. Sometimes you can find a function's roots by looking at its graph. Consider the graph of the function $f(x)=x^3-2x^2-5x+6$.

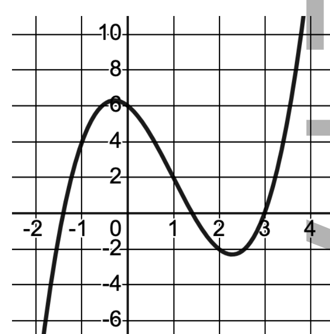


a. What are the function's roots?

b. Write the function in factored form.

The previous function is easy to write in factored form because each root is an integer. When a function's roots are not integers, you need another strategy to find their exact values.

9. Notice the graph of the function $f(x)=x^3-3x^2-2x+6$. In the graph, you can see one of the roots is 3, but you can only estimate the values of the other two roots.



a. Estimate the other 2 roots.

b. Since one root is 3, one of the polynomial's factors is $(x-3)$. Although you don't know the other roots, you can write the function in a partially factored form as $f(x)=(x-3)(\ \ ?\ \ )$. What is the degree of the missing expression?

c. The missing expression is $\left(x^2-2\right)$. You will learn how to find this later. How can you use this expression to find the remaining roots of the function?

10. Write the 3 roots of the previous function, $f(x)=x^3-3x^2-2x+6$.

Section 4

Imaginary Roots of Polynomials

Use this page for taking notes or anything else that helps you learn.

Rational Root Theorem

1. Find the roots of each quadratic shown.

a. $y = x^2 - 3x - 18$

b. $y = 6x^2 - 17x + 5$

c. Notice the roots and the original equations. Do you notice a connection between the polynomial's coefficients and its roots?

Rational Root Theorem: The possible rational roots of $y = \mathrm{A}x^n + \cdots + \mathrm{B}$ are $\frac{\text{factors of B}}{\text{factors of A}}$.

2. If a function has rational roots, the Rational Root Theorem shows which roots are possible. For example, the rational roots of "$y = 3x^2 + \cdots + 2$" could be $\frac{\text{factors of 2}}{\text{factors of 3}}$. One root could be $\frac{2}{3}$, but $\frac{3}{2}$ is not possible. Identify all possible rational roots for each function.

a. $y = x^2 + 4x - 9$

b. $y = 2x^2 - 7x + 13$

3. Consider the function below. List all possible rational roots of the function.

$f(x) = 3x^3 - 17x^2 - 12x - 10$

4. What are the possible rational roots of the function shown?

$f(x) = \mathrm{A}x^5 + \mathrm{B}x^2 + \mathrm{C}x + \mathrm{D}$

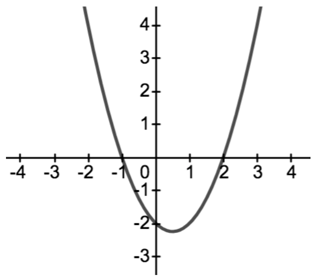
5. Circle all numbers that could be roots of the function $y = 6x^3 + 2x^2 - 7x + 15$.

-5 $\quad \frac{1}{3}$ $\quad -\frac{3}{2}$ $\quad \frac{5}{6}$ $\quad -\frac{1}{6}$ $\quad 15$

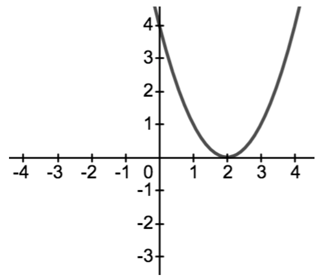
Polynomials with Imaginary Roots

10. In previous scenarios, you have seen that a polynomial function has as many roots as its degree. A 2nd degree function has 2 roots. They may be real or imaginary, but there are two. For each parabola below, identify the real roots and their multiplicity, and how many imaginary roots there are.

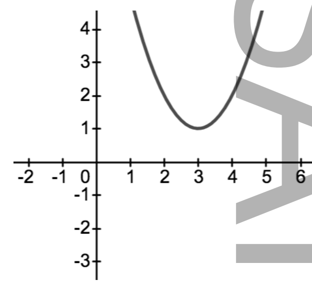
a.



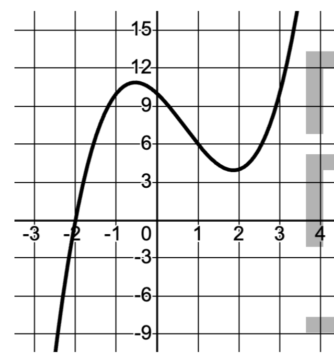
b.



c.



11. A cubic function has 3 roots. Consider the cubic polynomial shown.

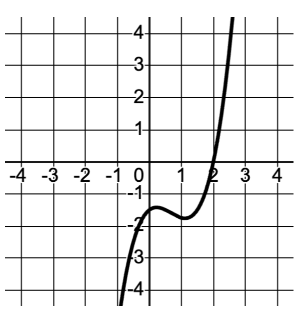


a. How many real roots does it have?

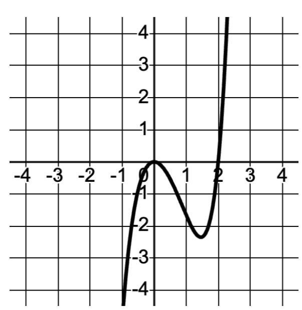
b. How many imaginary roots does it have?

12. Each graph below shows a 5th degree polynomial. For each graph, identify the real roots, the multiplicity of each real root, and how many imaginary roots there are.

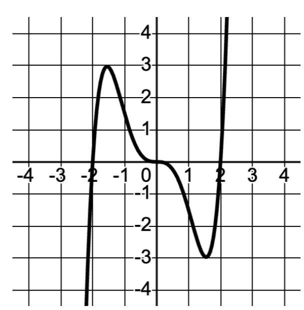
a.



b.



c.

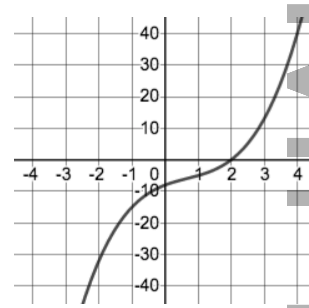


If you know some of the roots, you can use synthetic division to decrease the degree of the polynomial until the remaining polynomial is quadratic. The roots of the quadratic can be found by factoring, completing the square or using the quadratic formula.

13. A function is partially factored, as shown. Identify all of its roots.

$$f(x) = \left(2x^2 - x - 6\right)\left(x^2 - 6x + 13\right)$$

14. Consider the function $f(x) = x^3 - 2x^2 + 4x - 8$, shown in the graph. One of the roots is real and visible in the graph. Find the two remaining roots. Then, write the function in factored form, using only integer coefficients.



15. Use a graphing calculator to graph the function below. Use the graph to identify all obvious roots. Use synthetic division to find the remaining roots of the function.

$$f(x) = x^3 - 3x^2 - 20x - 6$$

16. Graph the function below on a graphing calculator. Use the graph to identify all obvious roots. Use synthetic division to find the remaining roots of the function.

$$f(x) = x^4 - 3x^3 - 2x^2 + 10x - 12$$

CONTENTS

Unit 6

Inverse, Square Root & Cube Root Functions

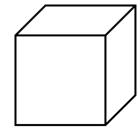
Section 1

Inverses of Functions

Use this page for taking notes or anything else that helps you learn.

Inverse Functions

1. If the edges of a cube are 2 inches long, what is the cube's volume?



2. To answer the previous question, you need to know that the volume of a cube depends on its edge length. The relationship between a cube's volume and its edge length is modeled by the cubic function, $V = L^3$, where L is the edge length. What is the volume of each cube described below?

   a. edge length = 4 in. b. edge length = 1 m c. edge length = $\frac{1}{2}$ cm

3. If a cube has a volume of 27 cm^3, how long is each edge of the cube?

4. To answer the previous question, you can reverse the operation of the cubic function. If the "Volume function" is written as $V = L^3$, reverse this equation to isolate L and create a "Length function."

   If $V = L^3$, then $L =$

5. An **inverse function** reverses the operations of a function. The inverse of $V = L^3$ is $L = \sqrt[3]{V}$. If a function adds 3 to each input, its inverse subtracts 3 from each input. If a function multiplies each input by 5, what does its inverse function do?

6. One way to find a function's inverse is to isolate the other variable. The inverse of $d = 2r$ is $r = \frac{d}{2}$. Isolate the input variable of each function to make its inverse.

   a. $G = b + 4$ b. $C = \pi d$ c. $y = \frac{x}{2} - 3$

Notation for Inverse Functions

7. Consider a simple function that multiplies each input by 2. The equation for this function can be written in many ways, depending on the variables you choose. Each equation below displays this function. Fill in the missing blanks to complete the last 3 functions.

a. $y = 2x$ b. $B = 2A$ c. $f(x) = 2$___ d. $f(n) = 2$___ e. output $= 2($________$)$

The inverse of $y = 2x$ is $x = \frac{y}{2}$. If you use $f(x)$ instead of y, the inverse looks more complex: $x = \frac{f(x)}{2}$. To avoid this complexity, there is a typical notation used for inverse functions.

When a function is written as $f(x)$, its inverse is $f^{-1}(x)$. The expression $f^{-1}(x)$ means "f inverse of x" or "the inverse of $f(x)$." Each function uses the variable x as its input. Note: $f^{-1}(x)$ does not mean $\frac{1}{f(x)}$.

Suppose $f(x)$ multiplies inputs by 2: $f(\text{input}) = 2(\text{input}) \rightarrow f(x) = 2x$

The inverse, $f^{-1}(x)$, divides inputs by 2: $\rightarrow f^{-1}(\text{input}) = \frac{\text{input}}{2} \rightarrow f^{-1}(x) = \frac{x}{2}$

8. Consider 2 simple functions.

a. If $f(x) = 3x$, what is $f^{-1}(x)$?

b. If $g(n) = \frac{n}{5}$, what is $g^{-1}(n)$?

9. Find the inverse of each function by isolating the input variable. After isolating x, switch x and y to make the function have a standard format, with x as the input variable. Show that it is the inverse function by replacing y with the notation $f^{-1}(x)$. The first one is done for you.

a. $f(x) = (x+2)^3$

replace $f(x)$ with y

$y = (x+2)^3$

isolate x

$\sqrt[3]{y} = x + 2$

$\sqrt[3]{y} - 2 = x$

$x = \sqrt[3]{y} - 2$

switch x and y

$y = \sqrt[3]{x} - 2$

replace y with $f^{-1}(x)$

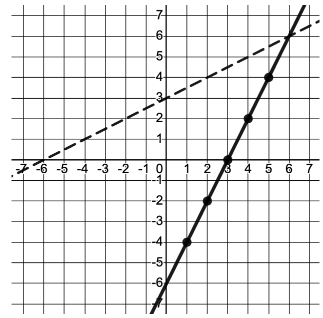
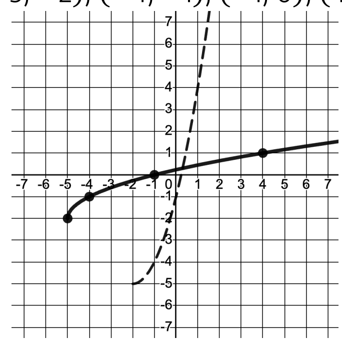
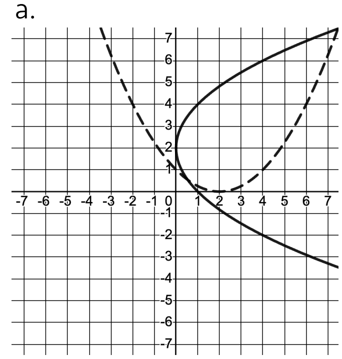
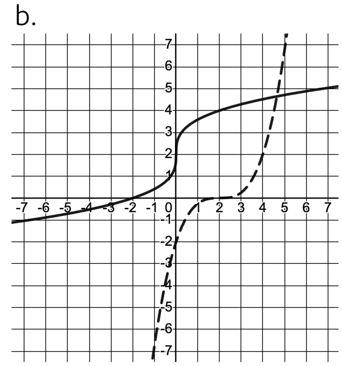
$f^{-1}(x) = \sqrt[3]{x} - 2$

b. $f(x) = 5x - 1$

c. $f(x) = \sqrt{x-6}$

Answer Key

1.	$2^3 \rightarrow 8 \text{ in}^3$
2.	a. 64 in^3 b. 1 m^3 c. $\frac{1}{8} \text{ cm}^3$
3.	3 cm
4.	$L = \sqrt[3]{V}$
5.	divide each input by 5
6.	a. $G - 4 = b$ b. $d = \frac{C}{\pi}$ c. $x = 2(y + 3)$
7.	c. $f(x) = 2x$ d. $f(n) = 2n$ e. output = 2(input)
8.	a. $f^{-1}(x) = \frac{x}{3}$ b. $g^{-1}(n) = 5n$
9.	b. $y + 1 = 5x \rightarrow \frac{y+1}{5} = x$ $f^{-1}(x) = \frac{x+1}{5}$ c. $y^2 = x - 6 \rightarrow y^2 + 6 = x$ $f^{-1}(x) = x^2 + 6$
10.	a. 1) subtract 3 2) raise to 3rd power 3) add 5 b. 1) subtract 5 2) take cube root 3) add 3 c. $f^{-1}(x) = \sqrt[3]{x-5} + 3$
11.	a. $f(x)$ operations: 1) subtract 2 2) multiply by 7 3) add 1 $f^{-1}(x)$ operations: 1) subtract 1 2) divide by 7 3) add 2 $f^{-1}(x) = \frac{x-1}{7} + 2$ b. $g(x)$ operations: 1) multiply by 3 2) subtract 4 3) take the square root $g^{-1}(x)$ operations: 1) raise to the 2nd power 2) add 4 3) divide by 3 $g^{-1}(x) = \frac{x^2+4}{3}$ c. $h(x)$ operations: 1) take the cube root 2) multiply by 5 3) add 10 $h^{-1}(x)$ operations: 1) subtract 10 2) divide by 5 3) raise to the 3rd power $h^{-1}(x) = \left(\frac{x-10}{5}\right)^3$ or $h^{-1}(x) = \left(\frac{x}{5} - 2\right)^3$
12.	5
13.	a. $f(4) = 3(4) + 1 = 12 + 1 \rightarrow f(4) = 13$ b. $f^{-1}(13) = \frac{13-1}{3} = \frac{12}{3} \rightarrow f^{-1}(13) = 4$
14.	Switch the x- and y-values $\rightarrow (-6, 13)$
15.	Switch the x- and y-values (5, 1), (9, 3) and (13, 5)

16.	 Switch the x- and y-values and plot the points: (1, −4), (2, −2), (3, 0), (4, 2), (5, 4)
17.	Plot the points: (−5, −2), (−4, −1), (−1, 0), (4, 1) 
18.	a.  b. 

Section 2

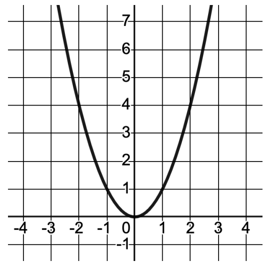
The Inverse of a Quadratic Function

Use this page for taking notes or anything else that helps you learn.

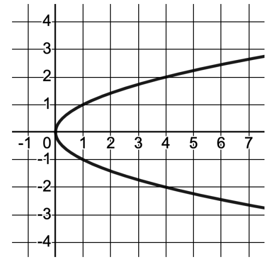
The Square Root Function

1. Consider the function $f(x) = x^2$. Its inverse is not a function.

the parabola



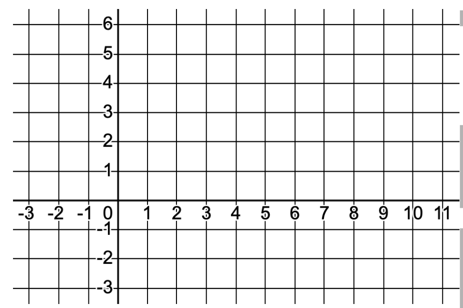
the parabola's inverse



Why is a parabola's inverse not a function?

2. A parabola has an inverse function if you restrict the domain of $y = x^2$ to its right side. The square root function $y = \sqrt{x}$ is the inverse of the right half of $y = x^2$.

Graph $y = \sqrt{x}$ using inputs that make integer outputs: (0, 1, 4, 9, etc...). Fill in the blanks below and use those 4 points to draw the square root function's curve.



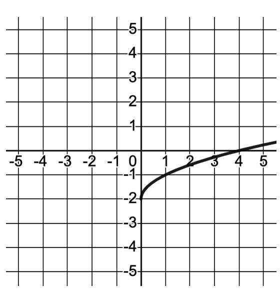
$\sqrt{0} =$ ___ $\sqrt{1} =$ ___ $\sqrt{4} =$ ___ $\sqrt{9} =$ ___

(0, ___) (1, ___) (4, ___) (9, ___)

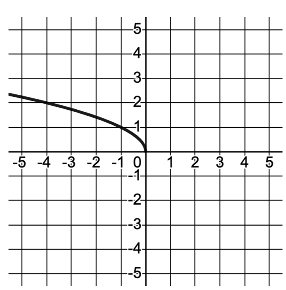
Applying Transformations to Square Root Functions

3. Each graph below shows a transformation of $y = \sqrt{x}$. Describe each one using the following words: horizontal, vertical, shift, stretch, compression, reflection.

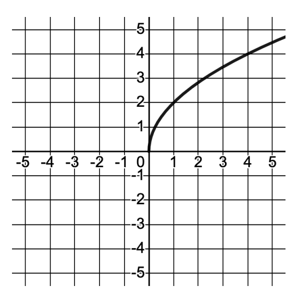
a.



b.



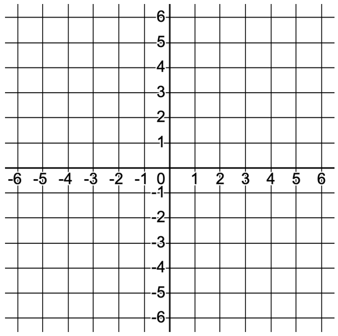
c.



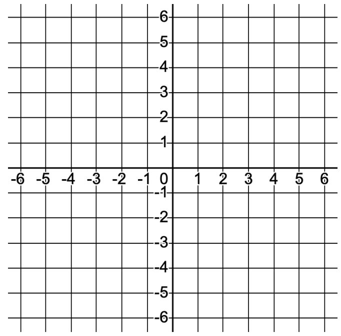
Graphing Square Root Functions

9. Before you practice graphing multiple transformations try to graph each transformation shown.

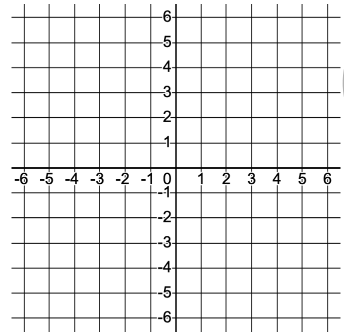
a. $y = \sqrt{x+4}$



b. $y = 3\sqrt{x}$

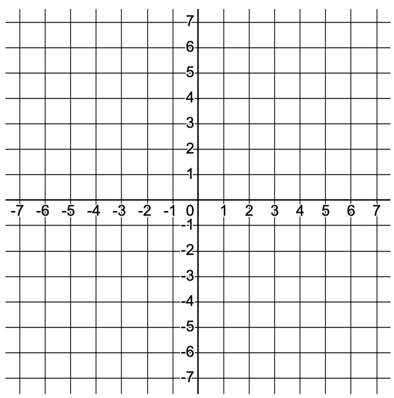


c. $y = \sqrt{2x}$

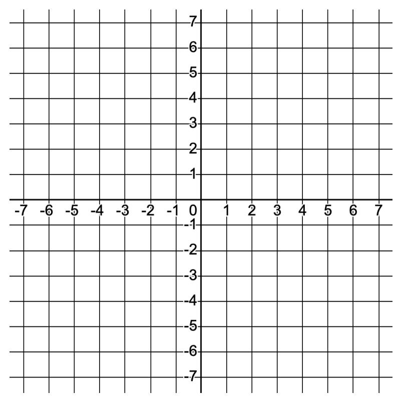


10. Graph each square root function. Plot the vertex and two more points before drawing the curve.

a. $f(x) = \sqrt{x+6} - 2$

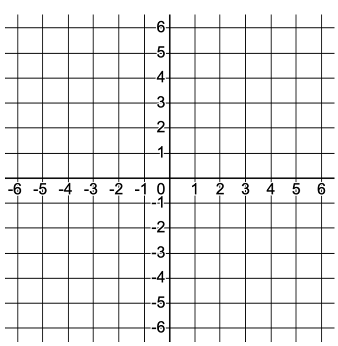


b. $f(x) = 3\sqrt{x} - 3$

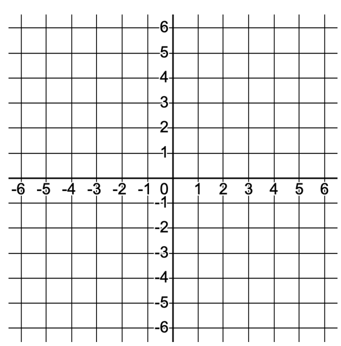


11. Before you practice graphing transformations that involve shifts, stretches and reflections, notice how reflections change the curve's direction. Try to graph each transformation shown.

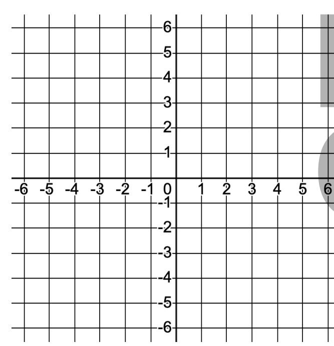
a. $y = -\sqrt{x}$



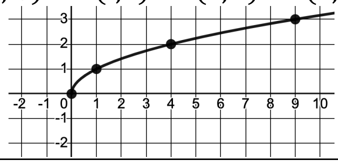
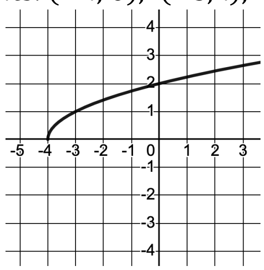
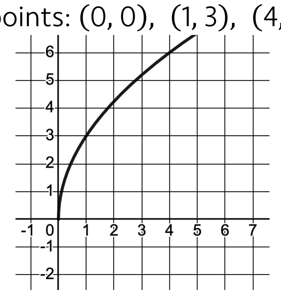
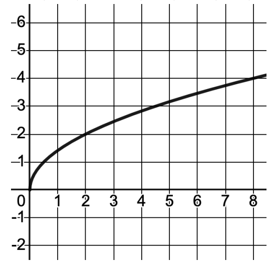
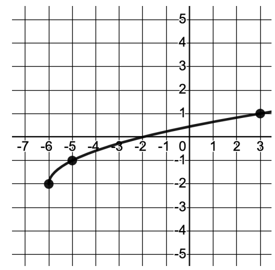
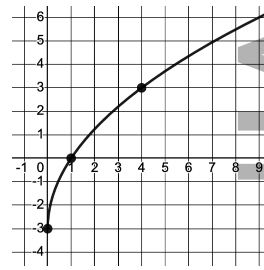
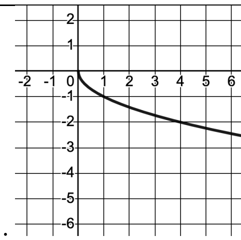
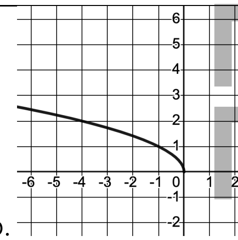
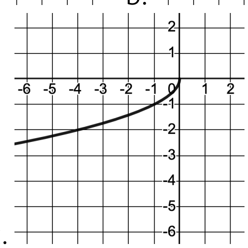
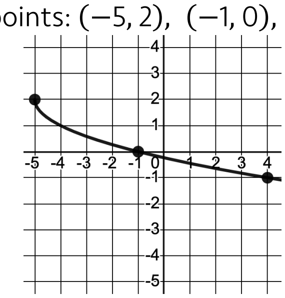
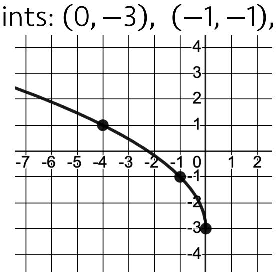
b. $y = \sqrt{-x}$



c. $y = -\sqrt{-x}$



Answer Key

1.	A parabola fails the horizontal line test, so its inverse is not a function. Its inverse fails the vertical line test.
2.	(0, 0) (1, 1) (4, 2) (9, 3) 
3.	a. down 2 b. reflect across y-axis c. vertical stretch by factor of 2 (or horizontal compression by a factor of 4)
4.	a. $(-5, 0)$ b. $(1, -9)$
5.	a. $(0, 4)$ b. $(-2, 0)$ c. (h, k)
6.	a. shift right 2: $(1, 1) \rightarrow (3, 1)$ shift down 6: $(3, 1) \rightarrow (3, -5)$ b. multiply y by 3: $(1, 1) \rightarrow (1, 3)$ shift down 4: $(1, 3) \rightarrow (1, -1)$ c. shift left 4: $(1, 1) \rightarrow (-3, 1)$ multiply y by 5: $(-3, 1) \rightarrow (-3, 5)$
7.	a. $y = \sqrt{x} + 2$ b. $y = \frac{1}{2}\sqrt{x}$ c. $y = \sqrt{-4x}$ d. $y = 2\sqrt{x+5}$
8.	b. $g(x) = \sqrt{-x} + 4$ c. $g(x) = -\sqrt{x+1}$ d. $g(x) = 4\sqrt{x}$
9.	a. shift left 4 3 points: $(-4, 0)$, $(-3, 1)$, $(0, 2)$  b. multiply y by 3 3 points: $(0, 0)$, $(1, 3)$, $(4, 6)$  c. multiply x-values by $\frac{1}{2}$ 5 points: $(0, 0)$, $(0.5, 1)$, $(2, 2)$, $(4.5, 3)$, $(8, 4)$ 
10.	a.  b. 
11.	 a.  b.  c.
12.	a. 3 points: $(-5, 2)$, $(-1, 0)$, $(4, -1)$  b. 3 points: $(0, -3)$, $(-1, -1)$, $(-4, 1)$ 

Section 3

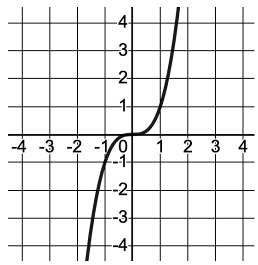
The Inverse of a Cubic Function

Use this page for taking notes or anything else that helps you learn.

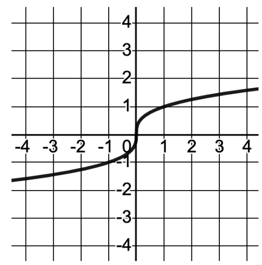
The Cube Root Function

1. The function $f(x) = x^3$ is shown below. Its inverse is also a function.

the cubic function

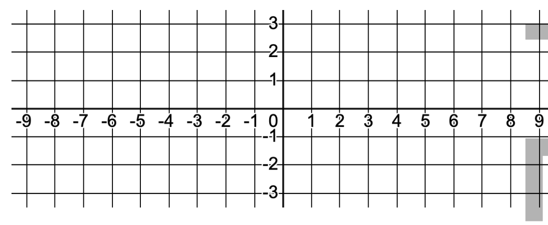


the inverse



What operation does the inverse apply to each input?

2. The inverse of the cubic function $y = x^3$ is the cube root function $y = \sqrt[3]{x}$. To graph $y = \sqrt[3]{x}$, use inputs that produce integer outputs: (0, 1, −1, 8, −8, etc...). Fill in the blanks below and use those 5 points to draw the cube root function's curve.



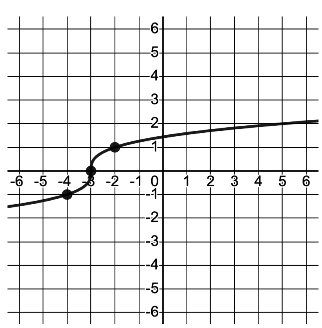
$\sqrt[3]{0} =$ ___ $\quad \sqrt[3]{1} =$ ___ $\quad \sqrt[3]{-1} =$ ___ $\quad \sqrt[3]{8} =$ ___ $\quad \sqrt[3]{-8} =$ ___

(0, ___) (1, ___) (−1, ___) (8, ___) (−8, ___)

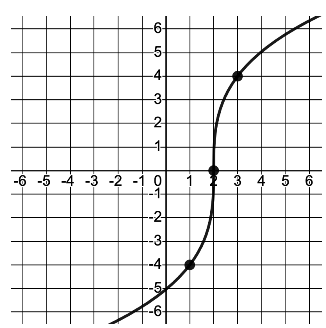
Applying Transformations to Cube Root Functions

3. Each graph below shows a transformation of the parent function $y = \sqrt[3]{x}$. Describe each one using the following words: horizontal, vertical, shift, stretch, compression, reflection.

a.



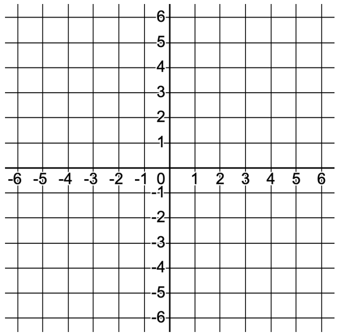
b.



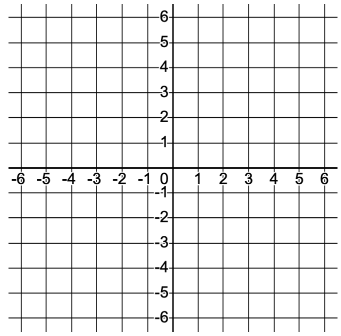
SAMPLE PAGES

4. Before you practice graphing transformations that involve shifts and stretches, first notice how reflections change the curve's direction. Try to graph each transformation shown.

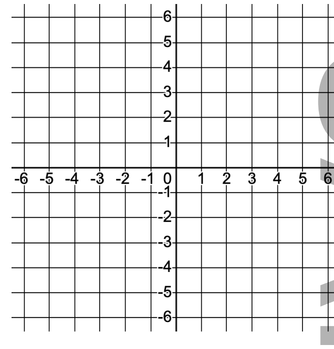
a. $y = -\sqrt[3]{x}$



b. $y = \sqrt[3]{-x}$



c. $y = -\sqrt[3]{-x}$



5. One important point on the graph of $y = \sqrt[3]{x}$ is $(0, 0)$. Like a cubic function, this is the cube root function's <u>inflection point</u>. Use what you know about transformations to identify the coordinates of each function's inflection point.

a. $f(x) = \sqrt[3]{x-7}$

b. $g(x) = \sqrt[3]{x+2} + 4$

6. Where is the inflection point of each function below?

a. $f(x) = 4\sqrt[3]{-x} - 8$

b. $g(x) = \sqrt[3]{0.5(x-3)}$

c. $h(x) = \mathrm{a}\sqrt[3]{\mathrm{b}(x-\mathrm{h})} + \mathrm{k}$

7. If $f(x)$ is defined by the equation $y = \sqrt[3]{x}$, write the new equation formed by each transformation.

a. $f(x) - 6$

b. $\frac{2}{3}f(x)$

c. $f(-2x)$

d. $-5f(x-1)$

8. Although you can apply both horizontal and vertical reflections to cube root functions, both of these reflections produce the same result. You can see this by simplifying each expression below.

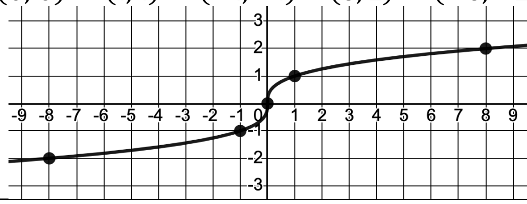
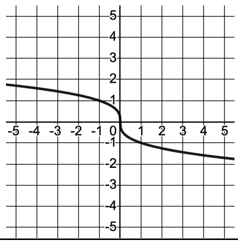
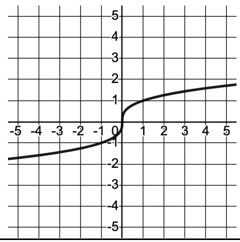
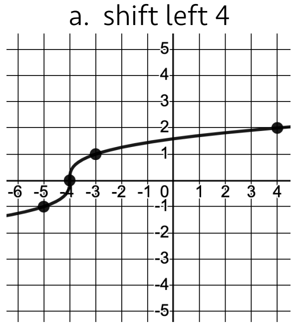
a. $\sqrt[3]{-1} =$

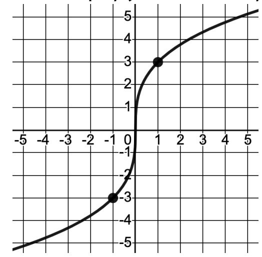
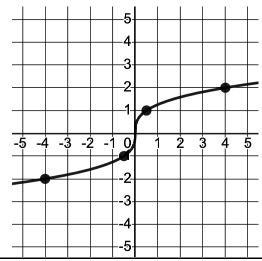
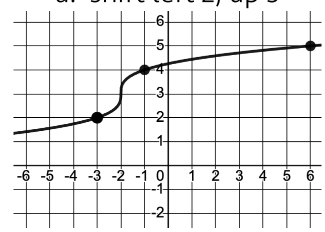
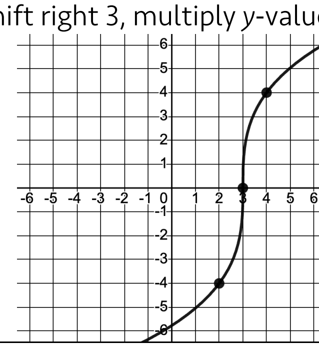
b. $\sqrt[3]{-x} =$

c. $y = \sqrt[3]{-x} \rightarrow y =$

9. If you reflect a cubic function across the y-axis, it looks the same as it does when you reflect it across the _______________, because the equation $y = \sqrt[3]{-x}$ can be rewritten as _______________.

Answer Key

1.	It finds the cube root of each input.
2.	(0, 0) (1, 1) (−1, −1) (8, 2) (−8, −2) 
3.	a. shift left 3 c. shift right 2, vertical stretch by factor of 4
4.	a and b.  c. 
5.	a. (7, 0) b. (−2, 4)
6.	a. (0, −8) b. (3, 0) c. (h, k)
7.	a. $y=\sqrt[3]{x}-6$ b. $y=\frac{2}{3}\sqrt[3]{x}$ c. $y=\sqrt[3]{-2x}$ d. $y=-5\sqrt[3]{x-1}$
8.	a. −1 b. $-\sqrt[3]{x}$ c. $y=-\sqrt[3]{x}$
9.	x-axis $y=-\sqrt[3]{x}$
10.	b. $y=\sqrt[3]{-x}+4$ c. $y=-\sqrt[3]{x+1}$ d. $y=\sqrt[3]{3x}$
11.	$y=\sqrt[3]{8x}$ → When x is replaced with $8x$, the function is a horizontally compressed by a factor of 8. $y=2\sqrt[3]{x}$ → vertical stretch by a factor of 2
12.	$y=\sqrt[3]{\frac{1}{27}x}$ → horiz. stretch by a factor of 27 $y=\sqrt[3]{\frac{1}{27}x} \rightarrow y=\frac{1}{3}\sqrt[3]{x}$ vertical compression by a factor of 3
13.	a. shift left 4 

	b. multiply y-values by 3  c. multiply x-values by $\frac{1}{2}$ 
14.	a. shift left 2, up 3  b. shift right 3, multiply y-values by 4 
15.	a. $f(-5)=0$ $f(-4)=1$ $f(-6)=-1$ $f(3)=2$ $f(-13)=-2$ $f(22)=3$ b. $g(0)=0$ $g(2)=1$ $g(-2)=-1$ $g(16)=2$ $g(-16)=-2$ $g(54)=3$

Section 4
Domains of Functions

Use this page for taking notes or anything else that helps you learn.

Domains of Square Root Functions

1. Some functions have restricted domains and others do not. Consider the function $y = 4x^3 - 7x + 1$. This polynomial's domain is $(-\infty, \infty)$. Polynomial functions have no domain restrictions because every real number input raised to an integer exponent produces a real number output. Three polynomials are shown. Sketch a simple graph of each function and identify one point on the graph.

   a. $y = x^2 - 4$
   b. $y = -(x + 2)^3 + 5$
   c. $y = x^3(x - 3)^2$

2. One type of function that has a domain restriction is a square root function. The domain of a square root function is the inputs that produce real outputs. Consider the square root function $y = \sqrt{x} - 5$. If x is negative, the outputs are imaginary, so the domain of $y = \sqrt{x} - 5$ is $x \geq 0$ or $[0, \infty)$.

   a. For the function $y = \sqrt{x - 2}$, the domain is $x - 2 \geq 0$, which can be rewritten as $x \geq$ ____.

   b. What is the domain of $y = \sqrt{x + 7} + 4$?

   c. What is the domain of $y = -2\sqrt[3]{5 - x}$?

3. Using interval notation, identify the domain of each function shown.

   a. $y = \sqrt{\frac{1}{3}x + 2} - 8$
   b. $y = 2\sqrt{6 - 2x}$

Domains of Rational Functions

7. In an earlier lesson, you learned that fractions can be undefined. When is a fraction undefined?

8. For which x-value(s) is each fraction undefined?

a. $\dfrac{x+7}{x-4}$ b. $\dfrac{x+11}{5x+2}$ c. $\dfrac{x^3+9x}{x^2-9}$ d. $\dfrac{6x-1}{x^2+8x+12}$

> The previous expressions are called rational expressions (fractions with a polynomial in the numerator and the denominator). Using this definition, the function below is called a **rational function**. A rational function's domain is all real number except those that make it undefined.

9. Consider the rational function below. Since the fraction is undefined if $x = 4$, the domain of the function is all real numbers except $x = 4$. Write this domain in interval notation.

$$y = \frac{x+7}{x-4}$$

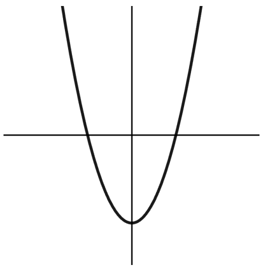
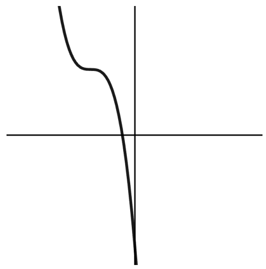
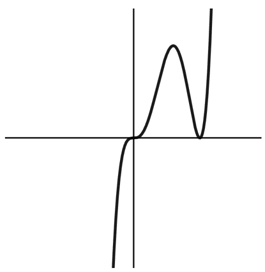
10. What is the domain of each function? Use interval notation.

a. $y = \dfrac{x-4}{x+7}$ b. $y = \dfrac{3x+1}{2x-12}$ c. $y = \dfrac{x+5}{4x-2}$

11. Using interval notation, write the domain of each function.

a. $y = \dfrac{x+4}{x^2+3x-10}$ b. $y = \dfrac{x^2-9x+20}{14x-2x^2}$

Answer Key

1.	a.  vertex: $(0,-4)$ b.  inflection pt: $(-2,5)$ c.  roots: $(0,0)$ and $(3,0)$
2.	a. $x \geq 2$ b. $x+7 \geq 0 \rightarrow x \geq -7$ c. $\sqrt[3]{x} \rightarrow$ all real numbers have a cube root
3.	a. $\frac{1}{3}x+2 \geq 0 \rightarrow x \geq -6 \quad [-6,\infty)$ b. $6-2x \geq 0 \rightarrow x \leq 3 \quad (-\infty,3]$
4.	a. $x^2 \leq 16 \rightarrow -4 \leq x \leq 4$ b. $x^2-4x-5 \geq 0 \rightarrow (x-5)(x+1) \geq 0$ $x \leq -1$ or $x \geq 5$
5.	a. $x^2-9 \geq 0 \rightarrow x^2 \geq 9$ domain: $x \leq -3$ or $x \geq 3$ b. $25-x^2 \geq 0 \rightarrow x^2 \leq 25$ domain: $-5 \leq x \leq 5$
6.	a. $x^2-5x-14 \geq 0 \rightarrow (x-7)(x+2) \geq 0$ domain: $(-\infty,-2] \cup [7,\infty)$ b. $4x-x^2 \geq 0 \rightarrow x(4-x) \geq 0$ domain: $[0,4]$
7.	A fraction is undefined when its denominator equals 0.
8.	a. $x=4$ b. $5x+2=0 \rightarrow x=-\frac{2}{5}$ c. $x^2-9=0 \rightarrow x=\pm 3$ d. $x^2+8x+12=0 \rightarrow (x+6)(x+2)=0$ $\rightarrow x=-6,-2$
9.	$(-\infty,4) \cup (4,\infty)$
10.	a. $x \neq -7 \rightarrow (-\infty,-7) \cup (-7,\infty)$ b. $x \neq 6 \rightarrow (-\infty,6) \cup (6,\infty)$ c. $x \neq \frac{1}{2} \rightarrow \left(-\infty,\frac{1}{2}\right) \cup \left(\frac{1}{2},\infty\right)$

11.	a. $x^2+3x-10 \neq 0 \rightarrow (x+5)(x-2)=0$ $\rightarrow x \neq -5, 2$ domain: $(-\infty,-5) \cup (-5,2) \cup (2,\infty)$ b. $2x(7-x) \neq 0 \rightarrow x \neq 0, 7$ domain: $(-\infty,0) \cup (0,7) \cup (7,\infty)$
12.	$x+3 \geq 0 \rightarrow x \geq -3$ and $x-2 \neq 0 \rightarrow x \neq 2$ The domain is $x \geq -3$, excluding $x=2$. domain: $[-3,2) \cup (2,\infty)$
13.	$x^2-6x+5 \geq 0 \rightarrow x \leq 1$ or $x \geq 5$ and $x-7 \neq 0 \rightarrow x \neq 7$ domain: $(-\infty,1] \cup [5,7) \cup (7,\infty)$
14.	a. $[-3,2) \cup (2,\infty)$ b. $(-\infty,1] \cup [5,7) \cup (7,\infty)$
15.	a. $[-6,-3) \cup (-3,\infty)$ b. $(-\infty,-4) \cup (-4,-2] \cup [0,\infty)$
16.	a. $3-x \geq 0 \rightarrow x \leq 3$ and $x+4 \neq 0 \rightarrow x \neq -4$ domain: $(-\infty,-4) \cup (-4,3]$ b. $x^2-3x \geq 0 \rightarrow x(x-3) \geq 0$ $\rightarrow x \leq 0$ or $x \geq 3$ and $x-5 \neq 0 \rightarrow x \neq 5$ domain: $(-\infty,0] \cup [3,5) \cup (5,\infty)$
17.	a.  1 3 b. $x=0 \rightarrow \frac{0-3}{0-1} \rightarrow 3 \rightarrow$ positive $x=2 \rightarrow \frac{2-3}{2-1} \rightarrow -1 \rightarrow$ negative $x=4 \rightarrow \frac{4-3}{4-1} \rightarrow 1 \rightarrow$ positive c. $(-\infty,1) \cup [3,\infty)$
18.	a.  -2 6 $x=-3 \rightarrow +$ $x=0 \rightarrow -$ $x=7 \rightarrow +$ solution: $(-\infty,-2) \cup [6,\infty)$ b.  -4 5 $x=-5 \rightarrow -$ $x=0 \rightarrow +$ $x=6 \rightarrow -$ solution: $[-4,5)$
19.	a. solve: $\frac{x+5}{x-7} \geq 0$ domain: $(-\infty,-5] \cup (7,\infty)$ b. solve: $\frac{8-x}{x+5} \geq 0$ domain: $(-3,8]$